Multi-Stage, Multi-Product Supply Chain Network Design with Solid Transportation and Fixed Costs

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Abstract
Supply Chain Management (SCM) is an optimal arrangement including the delivery of goods, services and information from supplier to buyer. The design of a transport network is one of the key areas of supply chain management. The design of this network provides great potential for reducing prices and improving the quality of services. Existence of transport vehicles and the transferring way of substances and products in a supply chain network is a very important issue. Transportation transfers goods between different parts of the chain. In addition to speed that is very effective on the efficiency of transportation in chain. So, for the first time, the solid transportation is considered in the multi-stage multi-product supply chain network design. Therefore a mathematical programming model is proposed and then the model is coded by Lingo Software to solve.

INTRODUCTION
In 1980s, organizations to achieve sustainable competitive advantage are mainly focused on systems such as just-in-time production, total quality management and so on. However, these competitive advantages lacked sustainability because of being imitated by competitors. In fact, effort for optimizing organizational processes regardless of external companies, especially suppliers and customers, seems to be useless and organizations who are working together toward common goals had better performance. It was here that the concept of the supply chain was born. Defines the supply chain as one of the most recent and best subjects that organizations by which attempt to create value for their shareholders and stakeholders [1].

The traditional transportation problem (TP) is a basic, useful and well-known optimization problem in operations research, which considers just two kinds of constraints, namely source constraint and destination constraint. The solid transportation problem (STP) is an extension of the TP, in which conveyances are considered in addition to source and destination. Generally, in most real world application and problems, a homogeneous product is carried from an origin to a destination by means of different types of conveyances (e.g., trucks, cargo flights, goods trains and ships). In other words, by considering a single type of conveyance, the STP is altered to a classical TP. In many practical applications, it is realistic to suppose that the amount that can be sent on any particular route bears a fixed charge for that route. Furthermore, when a route is altogether excluded, this can be expressed by limiting its capacity to zero. Such fixed transportation costs may also be applied to some production planning models [2]. The fixed charge transportation problem (FCTP) was initialized by Hirsch and Dantzig [3]. In an FCTP, the fixed cost is incurred for every route that is used in the solution, while the variable cost is proportional to the amount shipped. The objective is to find the combination of routes that minimizes the total variable and fixed costs while satisfying the supply and demand requirements of each origin and destination. Dissimilar to the transportation problem, the FCTP is more difficult to solve due to the fixed costs that result in discontinuities in the objective function and renders it unsolvable by the direct application of the transportation algorithm [4].

The existence of transportation vehicles and transfer of substances and products in a supply chain network is a very important issue. Transportation transfers goods between different parts of chain. In addition to speed of transportation that is effective on responsiveness and efficiency of supply chain, the type of transportation that is used by a company or a factory is also effective on inventory and location of facilities. The role of transport in
the competitive strategy of a new company reveals when company determines targets of customer.

Till now, none has considered the solid transportation in the Multi-stage, multi-product supply chain network design. Hence, for the first time, we present a new mathematical programming approach to solve the problem.

**Literature**

In a real-world application, it is often difficult to estimate the actual penalties (e.g., transportation cost, delivery time, quantity of goods delivered, under-used capacity and demands). Depending upon different aspects, they fluctuate due to uncertainty in judgment, lack of evidence, insufficient information, and the like. In recent years, the STP received much attention and many models and algorithms under both crisp and uncertain environments have been investigated.

Klose [5] showed that a particular case of the FCTP, namely the single source FCTP, is NP-hardness. This also proves the NP-hardness of the FCTP. Therefore, the computational time to obtain an exact solution is increased in a polynomial fashion, and it becomes very difficult to optimally solve the problem in reasonable time when the problem size increases. Adlakha and Kowsalski [6] proposed a simple heuristic algorithm for solving small fixed charge transportation problems. However, it is stated that the proposed algorithm is more time-consuming than the algorithms for solving a regular transportation problem. Besides, some other heuristics were proposed for solving fixed cost transportation problems by Gottlieb and Paulmann [7] and Sun et al [8].

On the other hand, Lixing and Yuan [9] proposed three mathematical models for two-criterion solid transportation with fixed costs in a probabilistic environment and used a Tabu search (TS) algorithm to solve the proposed models. As such, Lixing and Linzhong [10] solved these three mathematical models using hybrid intelligent algorithm, which is based on fuzzy simulation techniques and Tabu search algorithm. Melo et al. [11] examined previous studies in the field of designing supply chain network. They studied the basic characteristics of the problem of supply chain network such as the planning of supply chain strategies, location and design of the supply chain network with respect to the nature of capacity, demand and so on. They examined problem-solving methods such as reverse logistics and methods.

Bodhke et al. [12] used fuzzy planning techniques to solve solid transportation multi-objective problem. They used a special non-linear membership function (Hyperbolic and Exponential) to present the objective function in the fuzzy environment. As well as, Ojha et al. [13] examined transportation cost based on the amount of transportation in solid transportation and solved it with GA algorithm. Pandian and Natarajan [14] proposed zero-point method to find the optimal solution of solid transportation problem. Zhimiao and Jiuping [15] examined the application of type of multi-objective planning in solid transportation problem. To solve this kind of problems, they used GA, which is based on the simulation.

Also, Molla-Alizadeh-Zavardehi et al. [16] proposed a mathematical model for designing a two-stage supply chain network problem with fixed costs transportation. Compared with previous papers, they used high capacity for distribution centers. To solve the related model, they used GA and Artificial Immune System algorithms. Omar and Samir [17] and Chanas and Kuchta [18] discussed the solution algorithm for solving the transportation problem in a fuzzy environment. The entropy optimization in transportation models and other models is discussed in Kapur and Kesavan [19].

**MATERIAL AND METHODS**

**The proposed mathematical model**

The problem can be stated as a distribution and location problem, in which there are $S$ suppliers, $K$ plants, $J$ DCs, $I$ customers and $Q$, $H$ and $E$ conveyances in each stages (different modes of transport may be trucks, cargo flights, goods trains, ships, etc.). As shown in Fig 1, in the first stage, Each of the $S$ suppliers can ship raw materials to any of the $K$ plants using any of the $Q$ conveyances at a shipping cost per unit $t_{skq}$ (i.e., unit cost for shipping from supplier $s$ to plant $k$ by means of the $q$-th conveyance) plus a fixed cost $f_{skq}$ assumed for opening this route. Similarly, the products flow is determined in other stages. Consequently, the location of plants and DCs are determined. The objective is to determine, in which routes, plants and DCs are to be opened and the size of the shipment on those routes using conveyances in such a way that the total cost of the supply chain is minimized while satisfying the supply and shipment capacity constraints. The problem formulation is shown below.

![Figure 1. A simple network of Multi-stage, multi-product in supply chain network with solid transportation](http://sjmie.science-line.com/)
Indices

\[ S \quad (s=1,2,...,S) \text{ set of suppliers} \]
\[ K \quad (k=1,2,...,K) \text{ set of plants} \]
\[ J \quad (j=1,2,...,J) \text{ set of DCs} \]
\[ I \quad (i=1,2,...,I) \text{ set of customers} \]
\[ L \quad (l=1,2,...,L) \text{ set of products} \]
\[ Q \quad (q=1,2,...,Q) \text{ set of vehicles} \]
\[ H \quad (h=1,2,...,H) \text{ set of vehicles} \]
\[ E \quad (e=1,2,...,E) \text{ set of vehicles} \]

Parameters

\[ e_1 \quad \text{capacity number of vehicles 1} \]
\[ e_2 \quad \text{capacity number of vehicles 2} \]
\[ e_3 \quad \text{capacity number of vehicles 3} \]
\[ \text{cap}_{p_k} \quad \text{production capacity at the plant } k \]
\[ w_j \quad \text{annual throughput at DCs } j \]
\[ sp_s \quad \text{capacity of supplier } s \text{ for raw material} \]
\[ d_{ij} \quad \text{demand for product } l \text{ at customer } i \]
\[ v_{lk} \quad \text{unit production cost for product } l \text{ at plant } k \]
\[ r_{skq} \quad \text{unit transportation and purchasing cost for raw material from supplier } s \text{ to plant } k , \text{ by vehicles } q \]
\[ c_{kijh} \quad \text{unit transportation cost for product } l \text{ from plant } k \text{ to DCs } j, \text{ by vehicles } h \]
\[ c_{jile} \quad \text{unit transportation cost for product } l \text{ from DCs } j \text{ to customer } i, \text{ by vehicles } e \]
\[ r_{skq} \quad \text{fixed cost opening the route from supplier } s \text{ to the plant } k, \text{ by vehicles } q \]
\[ a_{kijh} \quad \text{fixed cost opening the route from plant } k \text{ to the DCs } j, \text{ by vehicles } h \]
\[ c_{jile} \quad \text{fixed cost opening the route from DCs } j \text{ to the customer } i, \text{ by vehicles } e \]
\[ g_k \quad \text{annual fixed cost for opening a plant } k \]
\[ p_k \quad \text{annual fixed cost for opening a DCs } j \]
\[ v_{ji} \quad \text{unit cost of throughput for product } l \text{ at site } j \]

Variables

\[ b_{skq} \quad \text{quantity of raw material shipped from supplier } s \text{ to plant } k \]
\[ f_{kijh} \quad \text{quantity of product } l \text{ shipped from plant } k \text{ to DCs } j \]
\[ y_{jil} \quad \text{quantity of product } l \text{ shipped from DCs } j \text{ to customer } i \]
\[ b_{skq} \quad 1 \text{ if } b_{skq} > 0 \text{ otherwise } 0 \]
\[ f_{kijh} \quad 1 \text{ if } \sum f_{kijh} > 0 \text{ otherwise } 0 \]
\[ y_{jil} \quad 1 \text{ if } \sum y_{jil} > 0 \text{ otherwise } 0 \]
\[ p_k \quad 1 \text{ if plant } k \text{ is opened, otherwise } 0 \]
\[ z_j \quad 1 \text{ if DCs } j \text{ is opened, otherwise } 0 \]

The problem can be formulated as follows:

\[
\text{Min } z = \sum_{s} \sum_{k} \sum_{e} t_{skq} b_{skq} + \sum_{k} \sum_{j} \sum_{h} a_{kijh} f_{kijh} + \sum_{j} \sum_{i} g_k y_{jil} + p_k + \sum_{j} z_j
\]

Subject to:

\[
\begin{align}
\sum_{s} \sum_{k} b_{skq} & \leq S_{pq} & \forall s, q \\
\sum_{k} \sum_{j} \sum_{h} f_{kijh} & \leq c_{pq} p_k & \forall k, q \\
\sum_{j} \sum_{i} y_{jil} & \leq w_j z_j & \forall j, l \\
\sum_{s} \sum_{k} b_{skq} & \leq e_s & \forall s, q \\
\sum_{k} \sum_{j} \sum_{h} f_{kijh} & \leq e_2 & \forall h \\
\sum_{j} \sum_{i} \sum_{h} y_{jil} & \leq e_2 & \forall e, i \\
\sum_{k} \sum_{j} \sum_{h} f_{kijh} & \geq \sum_{l} \sum_{j} \sum_{h} y_{jil} & \forall j, l \\
b_{skq} & = 0 & \text{if } b_{skq} = 0 & \forall s, q \\
b_{skq} & = 1 & \text{if } b_{skq} > 0 & \forall s, q \\
f_{kijh} & = 0 & \text{if } f_{kijh} = 0 & \forall k, j, h \\
f_{kijh} & = 1 & \text{if } f_{kijh} > 0 & \forall k, j, h \\
y_{jil} & = 0 & \text{if } y_{jil} = 0 & \forall j, i, e \\
y_{jil} & = 1 & \text{if } y_{jil} > 0 & \forall j, i, e \\
z_j & = \{0,1\} & \forall j \\
p_k & = \{0,1\} & \forall k \\
b_{skq} & \geq 0 & \forall s, k, q \\
f_{kijh} & \geq 0 & \forall k, j, l, h \\
y_{jil} & \geq 0 & \forall j, i, l, e \\
\end{align}
\]

RESULTS AND DISCUSSION

The objective of problem under consideration is to minimize the total cost supply chain, including transportation costs, fixed costs re-opening of the route, the cost of establishing the plants, the cost of distribution centers, production costs and the costs of storage. Constraint (2) states that the total output of each supplier for raw material are less than or equal to the capacity of that supplier. Constraint (3) states the total number of products sending from DCs to the plants less than or equal production capacity at the plants when it is established in return. Constraint (4) states the total number of products sending from DCs to customers less than or equal to the amount of the DCs for its establishment. Constraint (5) states the total amount of raw material sending from suppliers to plants is less than or equal with a capacity of Vehicles 1. Constraint (6) states total product submissions from plants to DCs is less than or equal to the capacity of means of Vehicles 2. Constraint (7) states total product submission from DCs to customers is less than or equal to the capacity of means Vehicles 3. Constraint (8) any customer can meet his request for each product from all distribution centers. Constraint (9) states the raw materials required for the production of product in the plants should be equal or more than the required items. Constraint (10) states total input of each distribution center for each product is equal to total output of DCs for the desired product. Constraint (11) - (14) imposes the integrality restriction on the decision variables. Constraints (16) - (18) impose the non-negativity restriction on decision variables.
Mathematical model to solve the problem using Lingo software

To test the accuracy of the proposed model, a small problem with the Lingo software is solved. The related parameters are given in Tables 1-3:

<table>
<thead>
<tr>
<th>set of suppliers</th>
<th>set of plants</th>
<th>set of DCs</th>
<th>set of products</th>
<th>set of customers</th>
<th>set of vehicles 1</th>
<th>set of vehicles 2</th>
<th>set of vehicles 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S=3</td>
<td>K=3</td>
<td>J=3</td>
<td>L=2</td>
<td>J=3</td>
<td>Q=2</td>
<td>N=2</td>
<td>E=2</td>
</tr>
</tbody>
</table>

### Table 2. The variable and fixed costs

<table>
<thead>
<tr>
<th>Stages</th>
<th>variable costs</th>
<th>fixed cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results are expected as follows:

### Table 3. The other characteristics

<table>
<thead>
<tr>
<th>annual fixed cost for opening a plant</th>
<th>p=500</th>
<th>700</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit production cost for product at plant</td>
<td>$\bar{p}$</td>
<td>$\bar{p}$</td>
<td>$\bar{p}$</td>
</tr>
<tr>
<td>annual fixed cost for opening a DCs</td>
<td>$\alpha$</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>unit cost of throughput for product at DCs</td>
<td>$\gamma$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>capacity of supplier</td>
<td>$Sp$</td>
<td>5000</td>
<td>4000</td>
</tr>
<tr>
<td>production capacity at the plants</td>
<td>$sp$</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td>annual throughput at DCs</td>
<td>$w$</td>
<td>2000</td>
<td>4000</td>
</tr>
<tr>
<td>demand for product</td>
<td>$D$</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>capacity number of vehicles 1</td>
<td>$v_1$</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>capacity number of vehicles 2</td>
<td>$v_2$</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>capacity number of vehicles 3</td>
<td>$v_3$</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

### CONCLUSION

In this paper, we studied the multi-stage multi-product supply chain network design problem with fixed-charge solid transportation for the first time. Therefore, a mathematical programming model is proposed and then the model is coded by Lingo Software to solve. The results indicate that the model is accurate and true. As a future work, the step fixed charge [20] can be considered in the transportation. Another direction is to work on other algorithms, such as Genetic Algorithm [21], and Differential Evolution [22].

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### REFERENCES


