



A Goal Attainment Technique for Solving the Reliable Location and Allocation Problem

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Abstract

In this paper, we develop a multi-objective programming approach for a reliable supply chain design. Decisions include locating a number of facilities among a finite set of potential sites and allocating task assignment between facilities and customers to maximize profit. Demands, supplies, processing, transportation, storage and capacity expansion costs are all considered as the objective function. To develop the model, one additional objective function is added into the supply chain design problem. So, our multi-objective model includes (i) the minimization of the sum of current investment costs and the expected future processing, transportation, storage and capacity expansion costs, (ii) the maximization of the responsive level of the model for customers. Finally, we use the goal attainment technique to obtain the Pareto-optimal solutions that can be used for decision-making.

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INTRODUCTION

The problem of supply chain network design involves tactical decisions, which refers to the supply chain configuration. As the infrastructure in supply chain management, it has long-lasting effects on tactical and operational decisions of the company.

In the long-term it is very likely that any components of a system face with failure and may be impaired. However, in the classical design of the facility location models, this point has been generally arisen less attention. In fact, in most papers it is presumed that the systems will always work correctly. And facilities are located in a way that never seems to suffer any failure. However, in the real world, facilities are subject to failure. For example, bad weather, human error, war and other situations can cause our facilities to be broken. This requires shows the development of more efficient placement models that

take into account the possibility of damage to facilities. It is noteworthy that the few article that have taken the damage of the facility location as granted, have very simple models that are seems far from the reality.

On the other hand, it is worth mentioning that in today's competitive world, while taking into account the decisions and costs associated with the establishment of integrated inventory and transportation facilities in the distribution chain design, can dramatically reduce costs and increase customer satisfaction. The costs of each of these three key elements (the facilities, inventory and handling) are convertible into another.

For example, there is a relationship between the costs of facilities and transportation costs. That way, no matter how much distribution centers and naturally higher deployment costs are; the transportation costs among distribution centers and customers are becoming less.



Similarly, inventory costs and their deployments are associated with one other. Thus, taking into account the integration costs of these three elements in a model of effective agents can bring out significant benefits. But it should be noted that, in literature of distribution chains, commonly the facility location decisions (which are strategic in nature) and inventory decisions (which are tactical decisions) are considered separately and apart. This is because the relationship between tactical and strategic elements in the distribution chain optimization model will be ignored. Given that, it is only recently that researchers have considered the integrate models and inventory location models placement, development of these models are very important.

The first objective function of our proposed model is the minimization of the sum of the costs, and the other one is the maximization of the responsive level of the customers. Since the expected total cost, and the responsive level are in conflict with each other, it is proposed to set up a multi-objective design problem whose solution will be a set of Pareto-optimal possible design alternatives representing the trade-off among different objectives rather than a unique solution. To the best of our knowledge, only ϵ -constraint method [12] and fuzzy optimization [11] have been used to solve multi-objective SC design models. We use the goal attainment technique; see [10] for details, to solve the resulting multi-objective problem.

Literature survey

Researchers have made significant progress on facility location and allocation problems with respect to problem representations and solutions. Revelle and Laporte [8] reviewed literature regarding location problems, and described formal statements of the problems with different objectives, with multiple products and machines, and with spatial interactions.

Laporte, et al [6] considered a class of capacitated facility location problems in which customer demands are stochastic. The problem was formulated as a stochastic integer linear program, with binary variables in the first stage and continuous variables in the second stage. Albareda-Sambola et al. [7] addressed different modeling aspects of the capacity- and distance-constrained plant location problem. They presented a tabu search algorithm for optimal or near-optimal solutions. Amiri [5] addressed the distribution network design problem in a supply chain system that involves location production plants and distribution warehouses, and determining the best strategy for distributing the product from the plant to the warehouses and from warehouses to customers.

He presented an efficient heuristic solution procedure to select the optimum number, locations and capacities of plants and warehouses to open so that all customer demand is satisfied at minimum total costs of warehouses and plants. Snyder and Daskin [2] introduce several models, based on classical facility location problem, in which facilities may fail with a given probability. They minimize weighted sum of two objectives, one of which is a classical objective (ignoring distributions) and the other of which is the expected cost after accounting for distribution.

Customers are assigned to several facilities, one of which is the primary facility that serves it under normal circumstances, one of which serves it if the primary fails, and so on. Berman, et al [1] consider structural properties of a model that is less computationally tractable than Snyder and Daskin's but more general. They assume that customers do not know in advance which facilities are operational and must travel from facility to facility in search of a working site.

Reliable facility location models are related to network reliability theory (Shier [3] and Shooman [4]), which attempts to calculate or maximize the probability that a network remains connected after random link failures. It is also related to literature on facility location with congestion, in which facility are sometimes unavailable due to excess demand (rather than to facility disruptions).

Problem Definition

Since strategic decisions had lasted for a considerable time and facilities that are now deployed is expected to be used for a long period of time, and also due to the high investments associated with these decisions, the appropriate configuration of supply chain network is important. The main purpose of model in this chapter is strategic decisions (to determine the location of facilities) and tactical decisions (the rate of material flow between network components). Here, the local factories, distribution centers, volume flow among the facilities, the primary allocation, distribution centers and customer support are determined. Facilities were also constructed with limited capacity and the capacity for constructed facilities is considered as a variable.

Overview of the model is focused on minimizing the costs. That these costs include fixed costs of the facility, costs of transport between network components, shipping fees and the costs of keeping orders. It is also looks for the exact location of each facility through the facility potential destinations and the optimal flow rate between network-components are defines in the manner that will minimizes the cost of transport among them all.

Network discussed in this section, is a fabric forward logistics network. Every customer will satisfy all their demands from a single distribution center. In this issue two types of distribution centers will be addressed; the facilities in the first set never fail, while all of the facilities in the second set fail independently with the same probability, q . The first set is called unbroken, while the second is called broken. That in comparison, the establishment cost of the latter is higher. Since the distribution centers are broken, in addition to the primary allocation for each customer, it is necessary to consider the "allocation supporting center" for each customer. It is worth mentioning that, if in the primary allocation phase a specific customer has been assigned to a broken distribution center, he no longer needs an allocated support center, on the other hand if in the primary allocation phase a specific customer has been assigned to a unbroken distribution center, he needs an allocated support center.

Model Assumptions

- The problem is for a single-product model.
- It is for a single periodic model.
- The facility capacity (production centers and distribution centers) is limited. (The raw material suppliers have no capacity constraints).
- A different capacity level for each facilitator exists in any potential place.
- Suppliers and customers have fixed and known locations.
- Potential locations for the establishment of factories and distribution centers are known and discrete.
- Number of facilities to be constructed is not predetermined.
- Only the material flow between two successive surface layers of the network can be established. Also there is no relationship between the facilities of a layer.
- Customer demands are fixed and known.
- Each customer shall be responded by only one distribution center.
- Failure probability of the distribution centers (per cent when the distribution center is damaged or does not serve are different) are independent of each other.
- It should be noted that when a distribution center does not operate, it has no harmful effects on other distribution centers.

The Outcome of the Model:

- Find the optimal location of facilities (production centers and distribution centers) and allocate it to each facility.

- Allocation of customers to distribution centers in the primary allocation and the allocation of their support center.
- Select the suppliers and amount of the needed raw materials for production to each factory.
- Optimal flow between all of the facilities that are linked together in the entire network.
- Construction of the facility will determine the optimal capacity.

Objective Functions:

The first objective function: (To minimize the costs) = (Fixed costs of facility construction) + (Costs of transportation and Purchase of materials) + (Storage costs)

The second objective function: (To maximize of responsive level in the primary allocation)

1.1. Constraints:

Total demand in the network must be answered.

There should be a balance between input and output in all network layers.

The constraint of capacity for all facilities should be considered.

Logical constraints relating to capacity levels in the potential points, and logical constraints related to the primary allocation, and allocation of support centers should be met.

Positivity and binary constraints of the used variables in the model should be considered.

Model formulation

The following notation is used in the formulation of the model.

SU	index set of potential supplier sites
I	index set of potential plant sites
J	index set of potential distribution center sites
K	index set of fixed customer zones
N	index set of capacity levels available to the potential facilities
d_k	demand of customer zone k
f_i^n	fixed cost to construct plant with capacity level n at site i
$o_j^{n,u}$	fixed cost to construct distribution center with capacity level n at site i (if the facility is broken)
$o_j^{n,r}$	fixed cost to construct distribution center with capacity level n at site i (if the facility is unbroken)
$Cx_{su_{ui}}$	cost of transportation and purchase one unit raw material to plant i from supplier su
Cx_{ij}	cost of transportation and purchase one unit product to distribution center j from plant i

CU_{jk}^p	cost of transportation and purchase one unit product to customer zone k from distribution center j (in the primary allocation)	X_{ij}	= shipment from plant at site i to distribution center at site j
CU_{jk}^s	cost of transportation and purchase one unit product to customer zone k from distribution center j (in the supporting allocation)	U_{jk}	= demand of customer zone k delivered from distribution center at site j
q	probability of failure of distribution centers	w_i^n	$= \begin{cases} 1 & \text{if a plant with capacity level n is located at site i} \\ 0 & \text{otherwise} \end{cases}$
h_j	cost of storage one unit of product in the distribution center j	$y_j^{n,u}$	$= \begin{cases} 1 & \text{if a broken distribution center with capacity level n is located at site j} \\ 0 & \text{otherwise} \end{cases}$
$\Delta F1$	the minimum rate of the responsive level in the primary allocation	$y_j^{n,r}$	$= \begin{cases} 1 & \text{if a unbroken distribution center with capacity level n is located at site j} \\ 0 & \text{otherwise} \end{cases}$
$\Delta F2$	the minimum rate of the responsive level in the supporting allocation	D_{jk}^p	$= \begin{cases} 1 & \text{if the demand of customer at site k will be determined as primary allocation} \\ 0 & \text{otherwise} \end{cases}$
caw_i^n	capacity with level n for the potential plant at site i		
cay_j^n	capacity with level n for the potential distribution center at site j		

The decision variables are:

In term of the above notation, the problem can be formulated as follows:

Xsu_{sui} = shipment from supplier su to plant at site i

$$f_1 = \min \sum_{i \in I} \sum_{n \in N} w_i^n * f_i^n + \sum_{j \in J} \sum_{n \in N} y_j^{n,u} * o_j^{n,u} + \sum_{j \in J} \sum_{n \in N} y_j^{n,r} * o_j^{n,r} + \sum_{su \in SU} \sum_{i \in I} Xsu_{sui} * Cx_{sui} + \sum_{i \in I} \sum_{j \in J} X_{ij} * Cx_{ij} \\ + \sum_{j \in J} \sum_{k \in K} q * D_{jk}^s * CU_{jk}^s * d_k + \sum_{j \in J} \sum_{k \in K} (1 - q) * D_{jk}^p * CU_{jk}^p - \sum_{j \in J} \sum_{k \in K} q * D_{jk}^s * (CU_{jk}^s - CU_{jk}^p) * d_k \\ + \sum_{j \in J} \sum_{k \in K} h_j * (D_{jk}^s - D_{jk}^p) * d_k$$

$$f_2 = \max \sum_{j \in J} \sum_{k \in K} D_{jk}^p * d_k \quad (2)$$

Subject to:

$$\sum_{su \in SU} Xsu_{sui} = \sum_{j \in J} X_{ij} \quad \forall i \in I \quad (3)$$

$$\sum_{i \in I} X_{ij} = \sum_{k \in K} (D_{jk}^p + D_{jk}^s - D_{jk}^s) * d_k \quad \forall j \in J \quad (4)$$

$$\sum_{j \in J} \sum_{k \in K} D_{jk}^p * d_k \geq \Delta F1 * \sum_{k \in K} d_k \quad (5)$$

$$\sum_{j \in J} \sum_{k \in K} D_{jk}^s * d_k \geq \Delta F2 * \sum_{j \in J} \sum_{k \in K} D_{jk}^p * d_k \quad (6)$$

$$\sum_{su \in SU} Xsu_{sui} \leq \sum_{n \in N} w_i^n * caw_i^n \quad \forall i \in I \quad (7)$$

$$\sum_{j \in J} X_{ij} \leq \sum_{n \in N} w_i^n * caw_i^n \quad \forall i \in I \quad (8)$$

$$\sum_{i \in I} X_{ij} \leq \sum_{n \in N} (cay_j^{n,r} * y_j^{n,r} + cay_j^{n,u} * y_j^{n,u}) \quad \forall j \in J \quad (9)$$

$$\sum_{k \in K} (D_{jk}^p + D_{jk}^b - D_{jk}^s) * d_k \leq \sum_{n \in N} (cay_j^{n,r} * y_j^{n,r} + cay_j^{n,u} * y_j^{n,u}) \quad \forall j \in J \quad (10)$$

$$\sum_{j \in J} D_{jk}^p \leq 1 \quad \forall k \in K \quad (11)$$

$$\sum_{j \in J} D_{jk}^b \leq 1 \quad \forall k \in K \quad (12)$$

$$\sum_{j \in J} D_{jk}^b \leq \sum_{j \in J} D_{jk}^p \quad \forall k \in K \quad (13)$$

$$D_{jk}^s \leq D_{jk}^p, \quad D_{jk}^s \leq D_{jk}^b \quad \forall j \in J, k \in K \quad (14)$$

$$D_{jk}^p \leq \sum_{n \in N} (y_j^{n,r} + y_j^{n,u}) \quad \forall j \in J, k \in K \quad (15)$$

$$D_{jk}^b \leq \sum_{n \in N} y_j^{n,r} \quad \forall j \in J, k \in K \quad (16)$$

$$\sum_{n \in N} (y_j^{n,r} + y_j^{n,u}) \leq 1 \quad \forall j \in J \quad (17)$$

$$\sum_{n \in N} \sum_{j \in J} y_j^{n,r} \geq 1 \quad (18)$$

$$\sum_{n \in N} w_i^n \leq 1 \quad \forall i \in I \quad (19)$$

$$Xsu_{sui}, X_{ij} \geq 0 \quad \forall i \in I, j \in J, su \in SU \quad (20)$$

$$w_i^n, D_{jk}^p, D_{jk}^b, D_{jk}^s, y_j^{n,r}, y_j^{n,u} \in \{0, 1\} \quad \forall i \in I, j \in J, k \in K, n \in N \quad (21)$$

The first objective function (1) minimizes total costs made of: the costs to serve the demands of customers from the distribution centers, the costs of shipments from the suppliers to the plants and from the plants to the distribution centers, and the costs associated with located facilities and storage. The second objective function (2) maximizes the responsive level in the primary allocation. Constraint sets (3) and (4) guarantee that the inputs and outputs in all network layers should be balanced. Constraint (5) and (6) ensure that the responsive level in the both primary and supporting allocations must be greater than or equal to specific amount. Constraint sets (7) – (10) represent the capacity restrictions of the facilities.

Constraint (11) and (12) ensure that a customer can be assigned at most one distribution center in the both primary and supporting allocation. Constraint set (13) represents that each customer can be assigned as a supporting allocation if it has assigned as a primary allocation before. Constraint set (14) ensures that if a customer assigned to a distribution center as both primary and supporting allocation, D_{jk}^s (regarding to f_1)

must be equal to 1. Constraint set (15) ensures that the primary allocation can be chosen from unbroken or broken distribution centers and the supporting allocation must be chosen from unbroken distribution centers. Constraint sets (17) and (18) ensure that a distribution center and a plant, respectively, can be assigned at most a capacity level. Constraint set (18) guarantee that at least one unbroken distribution center should be located. Constraint sets (20) enforce the non-negativity restrictions on the corresponding decision variables and constraint set (21) enforces the integrality restrictions on the binary variables.

Goal attainment technique

We use the goal attainment technique, which is a variation of goal programming technique, to solve the multi-objective problem. Goal attainment method is one of the multi-objective techniques with priori articulation of preference information given. In this method, the preferred solution is sensitive to the goal vector and the weighting vector given by the decision-maker; the same as the goal programming technique. Goal attainment

method has fewer variables to work with and is a one-stage method, unlike interactive multi-objective techniques, so it will be computationally faster. Therefore, in terms of computational time, it is one of the best techniques to solve our problem.

This method required a bunch of ideas and weights. In the general term for our problem if t is the index for the objective function, then bf_t is showing the "tth" objective function of argument and wf_t is depicting the "tth"

Min GA

Subject to:

$$\begin{aligned} & \sum_{i \in I} \sum_{n \in N} w_i^n * f_i^n + \sum_{j \in J} \sum_{n \in N} y_j^{n,u} * o_j^{n,u} + \sum_{j \in J} \sum_{n \in N} y_j^{n,r} * o_j^{n,r} + \sum_{su \in SU} \sum_{i \in I} Xsu_{sui} * Cx_{sui} + \sum_{i \in I} \sum_{j \in J} X_{ij} * Cx_{ij} \\ & + \sum_{j \in J} \sum_{k \in K} q * D_{jk}^B * CU_{jk}^B * d_k + \sum_{j \in J} \sum_{k \in K} (1 - q) * D_{jk}^P * CU_{jk}^P - \sum_{j \in J} \sum_{k \in K} q * D_{jk}^S * (CU_{jk}^B - CU_{jk}^P) * d_k \\ & + \sum_{j \in J} \sum_{k \in K} h_j * (D_{jk}^B - D_{jk}^P) * d_k - Wf1 * GA \leq bf1 \end{aligned}$$

$$\sum_{j \in J} \sum_{k \in K} D_{jk}^P * d_k + wf2 * GA \geq bf2 \quad (25)$$

$$\sum_{su \in SU} Xsu_{sui} = \sum_{j \in J} X_{ij} \quad \forall i \in I \quad (26)$$

$$\sum_{i \in I} X_{ij} = \sum_{k \in K} (D_{jk}^P + D_{jk}^B - D_{jk}^S) * d_k \quad \forall j \in J \quad (27)$$

$$\sum_{j \in J} \sum_{k \in K} D_{jk}^P * d_k \geq \Delta F1 * \sum_{k \in K} d_k \quad (28)$$

$$\sum_{j \in J} \sum_{k \in K} D_{jk}^B * d_k \geq \Delta F2 * \sum_{j \in J} \sum_{k \in K} D_{jk}^B * d_k \quad (29)$$

$$\sum_{su \in SU} Xsu_{sui} \leq \sum_{n \in N} w_i^n * caw_i^n \quad \forall i \in I \quad (30)$$

$$\sum_{j \in J} X_{ij} \leq \sum_{n \in N} w_i^n * caw_i^n \quad \forall i \in I \quad (31)$$

$$\sum_{i \in I} X_{ij} \leq \sum_{n \in N} (cay_j^{n,r} * y_j^{n,r} + cay_j^{n,u} * y_j^{n,u}) \quad \forall j \in J \quad (32)$$

$$\sum_{k \in K} (D_{jk}^P + D_{jk}^B - D_{jk}^S) * d_k \leq \sum_{n \in N} (cay_j^{n,r} * y_j^{n,r} + cay_j^{n,u} * y_j^{n,u}) \quad \forall j \in J \quad (33)$$

$$\sum_{j \in J} D_{jk}^P \leq 1 \quad \forall k \in K \quad (34)$$

$$\sum_{j \in J} D_{jk}^B \leq 1 \quad \forall k \in K \quad (35)$$

$$\sum_{j \in J} D_{jk}^B \leq \sum_{j \in J} D_{jk}^P \quad \forall k \in K \quad (36)$$

$$D_{jk}^S \leq D_{jk}^P, \quad D_{jk}^S \leq D_{jk}^B \quad \forall j \in J, k \in K \quad (37)$$

objective function weight. In this technique wf_t is showing the relation between the failures to reach argument of bf_t for the "tth" objective function. As our problem is the minimizing, so the lower wf_t is closer to the objective function and it has higher importance to us. If " $wf_t = 0$ ", then the equivalent objective function must gain the amount of the argument or the related objective function amount. Our formulation related to this issue is as follow:

$$D_{jk}^p \leq \sum_{n \in N} (y_j^{n,r} + y_j^{n,u}) \quad \forall j \in J, k \in K \quad (38)$$

$$D_{jk}^{\bar{p}} \leq \sum_{n \in N} y_j^{n,r} \quad \forall j \in J, k \in K \quad (39)$$

$$\sum_{n \in N} (y_j^{n,r} + y_j^{n,u}) \leq 1 \quad \forall j \in J \quad (40)$$

$$\sum_{n \in N} \sum_{j \in J} y_j^{n,r} \geq 1 \quad (41)$$

$$\sum_{n \in N} w_i^n \leq 1 \quad \forall i \in I \quad (42)$$

$$X_{su} X_{ui}, X_{ij} \geq 0 \quad \forall i \in I, j \in J, su \in SU \quad (43)$$

$$w_i^n, D_{jk}^p, D_{jk}^{\bar{p}}, D_{jk}^s, y_j^{n,r}, y_j^{n,u} \in \{0, 1\} \quad \forall i \in I, j \in J, k \in K, n \in N \quad (44)$$

Lemma1. If $(XSU^*, X^*, W^*, Y^*, D^*)$ is Pareto-optimal for the initial problem, then there exists a bf_t, wf_t pair such that $(XSU^*, X^*, W^*, Y^*, D^*)$ is an optimal solution to the optimization goal attainment problem.

The optimal solution using this formulation is sensitive to bf and wf_t . Depending on the values for bf , it is possible that wf_t does not appreciably influence the optimal solution. Instead, the optimal solution can be determined by the nearest Pareto-optimal solution from bf . This might require that wf_t be varied parametrically to generate a set of Pareto-optimal

solutions. In the next section, we consider several pairs of bf and wf_t to generate different Pareto-optimal solutions.

Computational Result

The given values of the parameter model are derived from different data sets which have been presented by Pishvae et al. [9] and Amiri [5]. Table (1) is showing the overall changes of the parameters of the proposed model.

Table 1. Size matters designed to solve the proposed model.

Parameters	amounts	Parameters	amounts
d_k	Uniform (80, 250)	$cxsu$	Uniform (4,12)
f_i	Uniform (450000,800000)	cx	Uniform (4,12)
o_j	Uniform (250000,500000)	cu	Uniform (4,12)
$capw_i$	Uniform (400000, 1000000)	h_j	Uniform (100,200)
$capy_j$	Uniform (40000, 100000)	$\Delta F1$	Uniform (0.8,1)
q	Uniform (0.025,0.075)	$\Delta F2$	Uniform (0.8,1)

It is worth noting that, according to Pishvae et al [9], for the other parameters and considering the different capacity levels, ($\forall n \in N$); at first we will generate a random capacity from the range given in

Table (1). Then, according to its value the different levels of capacity will be determined by using the following relations.

$$caw_i^n = capw_i + \left(\frac{n-1}{N-1}\right) * capw_i \quad \forall n \in N \quad (45)$$

$$cay_j^n = capy_j + \left(\frac{n-1}{N-1}\right) * capy_j \quad \forall n \in N \quad (46)$$

$$f_i^n = f_i + 0.3 * f_i * \left(\frac{caw_i^n - caw_i^1}{caw_i^1}\right) \quad \forall n \in N \quad (47)$$

$$o_j^{n,u} = o_j + 0.3 * o_j * \left(\frac{cay_j^n - cay_j^1}{cay_j^1}\right) \quad \forall n \in N \quad (48)$$

$$o_j^{n,r} = o_j^{n,u} + 10 * q * o_j^{n,u} \quad \forall n \in N \quad (49)$$

To calculate the costs of purchase and transportation between the various volume levels, and as far as these costs are inter-related to factors such as the distance of the facilities from each other, to calculate the factor of the distance of each facility from each other we have to determine the exact location of candidate facilities in different layers of the network. Values for the parameters of the candidate's geographic coordinates (longitude and latitude of the candidates and the establishment of centers of demand) are provided in accordance with reference [5]. In this regards, our premise is that the

latitude and longitude coordinates of the candidate are a uniform distribution in a square between (0, 100). Then, the Euclidean distance among each candidate centers are calculated according its latitude and longitude points.

Finally, the shopping and transportation costs per unit of product are calculated from the formulas (50) to (52).

$$C_{xsu_{sui}} = c_{xsu} * d(su, i)$$

$$C_{xij} = c_x * d(i, j)$$

$$C_{ujk} = c_u * d(j, k)$$

$$\forall su \in SU, i \in I \quad (50)$$

$$\forall i \in I, j \in J \quad (51)$$

$$\forall j \in J, k \in K \quad (52)$$

In this section, to illustrate the applicability of the proposed model, we have produced three problem

categories in different sizes. According to the table (4-1) these sizes are including small, medium and large.

Table 2. Size matters designed to solve the proposed model.

size	index	Number of suppliers	Number of plants	Number of distribution centers	Number of customers	Number of capacity levels
		SU	I	J	K	N
Small	S	3	2	3	5	2
Medium	M	10	5	7	15	5
Large	L	20	15	20	40	10

Then by using Cplex we have solved each of them for five times, and their results in small scale (S1 to S5) are shown in Table (2)

Table 3. Result of the solution in small scale.

Size	Problem number	Criterion of objective function	Sum of the all costs f_1	Fixed costs	Transportation costs	Responsive level f_2
Small S	S-1	f_1	1.70E+06	1.20E+06	4.96E+05	597
		f_2	2.03E+06	1.20E+06	8.27E+05	795
	S-2	f_1	1.58E+06	1.14E+06	4.36E+05	555
		f_2	1.89E+06	1.14E+06	7.55E+05	708
	S-3	f_1	1.92E+06	1.26E+06	6.60E+05	607
		f_2	2.28E+06	1.44E+06	8.37E+05	809
	S-4	f_1	1.51E+06	1.08E+06	4.26E+05	507
		f_2	1.80E+06	1.08E+06	7.24E+05	776
	S-5	f_1	1.73E+06	1.24E+06	4.86E+05	589
		f_2	2.09E+06	1.39E+06	7.09E+05	798
	Medium	f_1	1.69E+06	1.18E+06	5.01E+05	571
		f_2	2.02E+06	1.25E+06	7.70E+05	777.2
	Standard deviation	f_1	2.53E+10	5.52E+09	8.88E+09	1662
		f_2	3.37E+10	2.44E+10	3.46E+09	1637.7

Other features of this table 3 is showing the optimal value for the cost of network and the response rates of it in a situation in which each objective function is considered as a single criterion objective function.

The objective function of the total cost of network and the objective function associated with response rates of network are respectively coded as F1 and F2. To explain the table data we can refer to the following two examples:

Row and column F1 are related to the problem of S1. It means that the figure 17005x106 is showing that the optimized costs of the network are accepted as criterion function. On the other hand, row and column F2 in our S1 problem is equals 795, which is showing the amount of response of the network as a criterion function.

Table 4. the average results of goal attainment technique for the proposed model in three size scales.

	bf1	bf2	Wf1	Wf2	f1	f2	GA
S	1.85E+06	7.00E+02	1.00E+00	5.00E-04	1.97E+06	6.93E+02	1.19E+05
	1.85E+06	7.00E+02	9.99E-01	1.00E-03	1.88E+06	6.93E+02	1.86E+08
	1.85E+06	7.00E+02	5.00E-04	1.00E+00	1.81E+06	5.88E+02	3.58E+11
	1.85E+06	7.00E+02	1.00E-04	1.00E+00	1.73E+06	5.12E+02	1.71E+12
	1.70E+06	5.50E+02	1.00E+00	5.00E-04	1.61E+06	5.58E+02	1.60E+04
	1.70E+06	5.50E+02	9.99E-01	1.00E-03	1.69E+06	5.67E+02	-1.70E+04
	1.70E+06	5.50E+02	5.00E-04	1.00E+00	1.69E+06	5.71E+02	-2.00E+07
	1.70E+06	5.50E+02	1.00E-04	1.00E+00	1.69E+06	5.71E+02	-1.00E+08
	2.00E+06	8.00E+02	1.00E+00	5.00E-04	2.02E+06	7.71E+02	5.80E+04
	2.00E+06	8.00E+02	9.99E-01	1.00E-03	2.02E+06	7.66E+02	3.40E+04
	2.00E+06	8.00E+02	5.00E-04	1.00E+00	2.01E+06	7.66E+02	2.00E+07
	2.00E+06	8.00E+02	1.00E-04	1.00E+00	2.01E+06	7.66E+02	1.00E+08
M	6.10E+06	2.20E+03	1.00E+00	5.00E-04	6.05E+06	2.00E+03	3.98E+05
	6.10E+06	2.20E+03	9.99E-01	1.00E-03	6.09E+06	2.00E+03	1.99E+05
	6.10E+06	2.20E+03	5.00E-04	1.00E+00	6.09E+06	2.00E+03	1.99E+02
	6.10E+06	2.20E+03	1.00E-04	1.00E+00	6.09E+06	2.00E+03	1.99E+02
	5.50E+06	1.50E+03	1.00E+00	5.00E-04	5.32E+06	1.50E+03	6.00E+03
	5.50E+06	1.50E+03	9.99E-01	1.00E-03	5.32E+06	1.50E+03	2.00E+03
	5.50E+06	1.50E+03	5.00E-04	1.00E+00	5.59E+06	1.52E+03	1.80E+08
	5.50E+06	1.50E+03	1.00E-04	1.00E+00	5.55E+06	1.51E+03	5.00E+08
	7.00E+06	3.00E+03	1.00E+00	5.00E-04	6.89E+06	2.51E+03	9.76E+05
	7.00E+06	3.00E+03	9.99E-01	1.00E-03	6.90E+06	2.51E+03	3.76E+09
	7.00E+06	3.00E+03	5.00E-04	1.00E+00	6.90E+06	2.51E03+	3.77E+06
	7.00E+06	3.00E+03	1.00E-04	1.00E+00	6.90E+06	2.51E+03	3.76E+06
B	1.70E+07	5.80E+03	1.00E+00	5.00E-04	1.74E+07	5.72E+03	4.20E+05
	1.70E+07	5.80E+03	9.99E-01	1.00E-03	1.74E+07	5.70E+03	4.20E+05
	1.70E+07	5.80E+03	5.00E-04	1.00E+00	1.74E+07	5.70E+03	8.40E+08
	1.70E+07	5.80E+03	1.00E-04	1.00E+00	1.74E+07	5.70E+03	4.20E+09
	1.50E+07	4.80E+03	1.00E+00	5.00E-04	1.59E+07	4.74E+03	1.30E+05
	1.50E+07	4.80E+03	9.99E-01	1.00E-03	1.57E+07	4.73E+03	6.51E+05
	1.50E+07	4.80E+03	5.00E-04	1.00E+00	1.51E+07	4.72E+03	1.80E+08
	1.50E+07	4.80E+03	1.00E-04	1.00E+00	1.50E+07	4.72E+03	8.10E+01
	2.00E+07	6.80E+03	1.00E+00	5.00E-04	1.93E+07	6.72E+03	1.68E+05
	2.00E+07	6.80E+03	9.99E-01	1.00E-03	1.95E+07	6.72E+03	8.40E+04
	2.00E+07	6.80E+03	5.00E-04	1.00E+00	1.95E+07	6.72E+03	8.40E+01
	2.00E+07	6.80E+03	1.00E-04	1.00E+00	1.96E+07	6.72E+03	8.00E+01

For analyzing the model structure, the optimized amount of expenses and the optimized amount of the responsive level are considered for each of the objective functions in different scale sizes separately.

According to the output table 3, moderate activity in the first objective function and in the small scale is a figure between 1.693 million and 2.021 million. So the amount of the argument "bf1" will be a random figure between 1.21 million and 2.57 million.

Note that the first figure is the average amount of the first objective function (when it is criterion) minus three times of the standard deviation of the first objective function. The second figure is the average amount of the first objective function (when the second function is criterion) plus the three times of standard deviation of the first objective function. It worth mentioning that, the argument amount of second objective function "bf2" will

be calculated randomly in the same way. Also, note that the amount of "wf1" and "wf2" are a random figure between 0.0001 and 0.9999 and the total amount of argument should be 1. The same way we will calculate other arguments and weights of other scale sizes. Table (4) is showing the average results of goal attainment technique for the proposed model in three size scales; small, medium, and big.

For analyzing the structure of the model, all the parameters of the model is kept constant. And response characteristics of model 1 (in medium size) are according to changing the demand values and the failure probabilities of the distribution centers. For this purpose, we used a measure of the cost function related to average value of the five problem of M-1 to M-5. The result of this analysis is shown in the diagram.

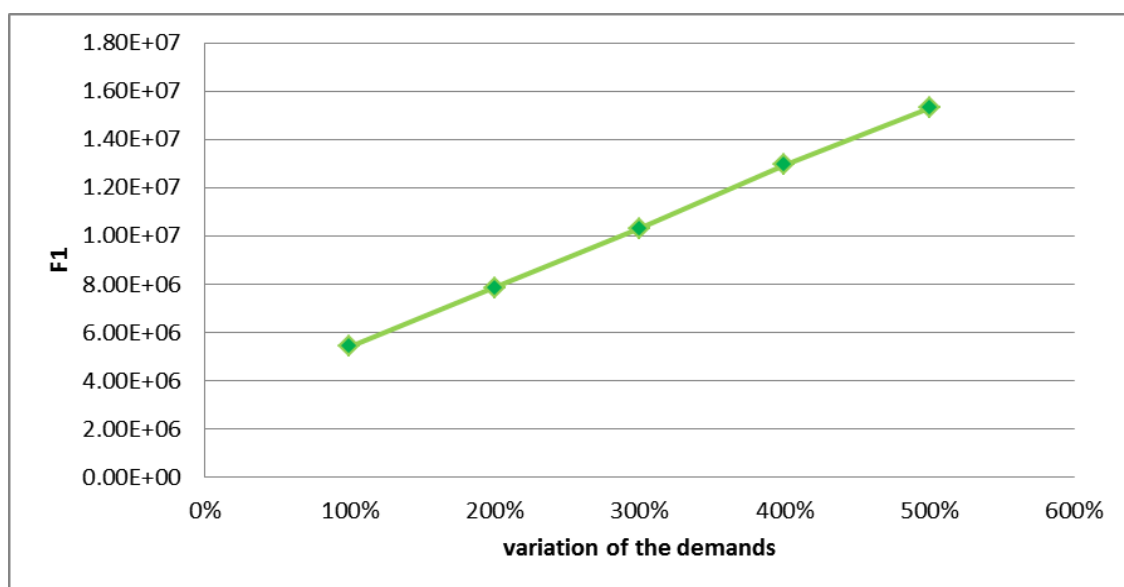


Fig 1. Objective function 1 under uncertain demands.

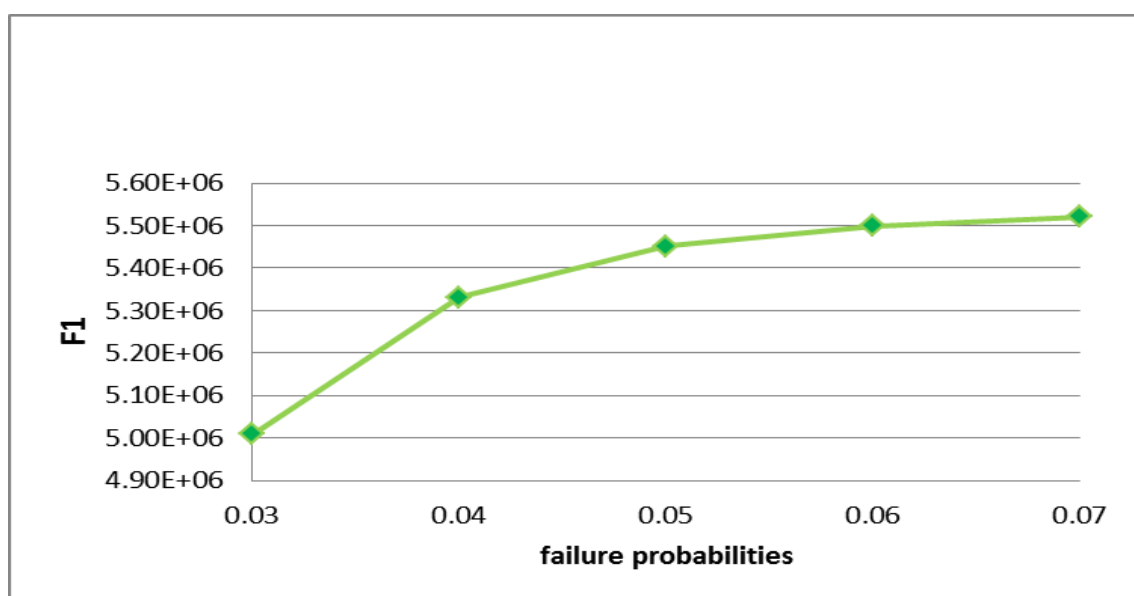


Fig 2. Objective function 1 under uncertain failure probabilities

Figure 1 shows, with increasing the demand, the system costs will face with increasing amounts. It can be realized from Figure 2, which by increasing the probability of failure of distribution centers; the system costs increase and absorb a fixed amount.

CONCLUSION

To cope with the issue of uncertain parameters in the supply chain network design problem, this paper proposes a multi objective model based on the recent extensions in reliability theory. The supply chain network considered in this paper is including customers, facilities and suppliers. Supply chain network design is one of the most important strategic decisions in supply chain management.

In this paper, network design decisions include determining the numbers, locations and capacities of facilities and the quantity of flow between them based on two goal; minimization of costs and maximization of responsive level of demand. The computational results of the numerical experiments imply that the goal attainment technique is effective to solve the model that can be used for decision-making.

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