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Step Fixed Charge Transportation Problems with Fuzzy Numbers

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Abstract

In this paper, we focused on a technique which obtains a good solution of the step fixed charge transportation problem, where both the fixed cost and the unit transportation cost from each origin to each destination, have been expressed as generalized trapezoidal fuzzy numbers. To do so, the present paper, first, tries to convert this problem into the fuzzy transportation problem, and then, tries to construct a fuzzy coefficient matrix to finding a good solution for step fixed charge transportation problem, by developing the earlier proposed formulae. Also, in addition to, being in line with these formulae tries to propose new formulae as alternative. To the best our knowledge, till now there is no exact algorithm for solving the fuzzy step fixed charge transportation problem, so, any heuristic method which provides a good solution should be considered useful and the current paper propose one. A numerical example is presented to illustrate the proposed method.

Original Article

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INTRODUCTION

Step fixed charge transportation problem (SFCTP) is a special version of the fixed charge transportation problem (FCTP). The SFCTP in its representation first was founded by Kowalski and Lev [16]. In SFCTP the fixed charge is incurred for each route that is used in the solution, along with the variable cost that is proportional to the amount shipped. Also, this cost structure causes the value of the objective function to behave like a step function [1]. In the case of the SFCTP due to the step function structure of the objective function, Kowalski and Lev [2] were dealing with a "NP- hard" problem. Since there was no algorithm for the SFCTP then, they tried to suggest two heuristic methods which provide a good solution as a useful method, by extending the method proposed by Balinski [3]. Altassan et al. [4], in addition to, first, being in line with them tried to suggest three other formulae, and then, tried to compare the performance of the new formulae with the earlier proposed formulae. Recently, El-Sherbiny [1] proposed the alternate mutation based artificial immune algorithm for SFCTP. They claimed that their algorithm solve both balanced and unbalanced SFCTP without introducing a dummy supplier or a dummy customer.

All of the aforementioned literatures briefly introduce the SFCTP concept in an effort to familiarize the reader with the underlying theory and then present the approaches with precise data to solve the SFCTP. In fact, for each possible transportation pattern in the real world, some or all the parameters are not only well-defined,

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precise data, but also vague or fuzzy data. Since the fuzzy set theory was proposed by zadeh in 1965 [5], we have been able to handle such data. The role of fuzzy sets in decision processes is best described in the original statements of Bellman and Zadeh [6]. Thus, decision processes are better described and solved by using fuzzy set theory, rather than precise approaches. To this end, the application of the fuzzy set theory to the linear programming and multi-criteria decision making problems. Chanas et al, [7] presented a fuzzy linear programming model to solve TP with fuzzy supply and demand values. Chanas and Kuchta [8] developed an algorithm to obtain the optimal solution based on type of TPs with fuzzy coefficients. Kaur and Kumar [9] tried to propose a new approach to solve a special type of fuzzy TPs by representing the transportation costs as generalized trapezoidal fuzzy numbers (GTFNs). Kaur and Kumar [14] proposed a new method to solve the fuzzy TPs, where transportation cost, availability and demand of the products are represented by the GTFNs.

As far as we know, with regard to solve the Fuzzy Step Fixed-Charge Transportation Problem (FSFCTP), no research has been done. Therefore, any method which provides a good solution for it will be distinguished. In order to, the present paper, first, tries to convert the FSFCTP into the fuzzy transportation problem (FTP) by developing the Kowalski's formulae in fuzzy environment. This becomes a linear version of the FSFCTP for the next stage, and then, tries to obtain a fuzzy initial basic solution and optimal solution both of the linear version of the FSFCTP by using one of the well-known methods, such as the generalized fuzzy north-west corner method, the generalized fuzzy least-cost method, the generalized fuzzy Vogel's approximation method and the fuzzy modified distribution method [9]. We also propose several other formulae for finding a fuzzy good initial solution in FSFCTP, and show that the result of new formulae are better than the generalized Kowalski's formulae, by comparing the fuzzy value objective function which is obtained by using these formulae. It can be an important advantage of the proposed method.

Most of the literatures on the FTP topic are only concerned with the normal fuzzy numbers instead of the generalized fuzzy numbers. They, first, try to convert the generalized fuzzy numbers into the normal fuzzy numbers by using the normalization process and then try to solve the real life problems by considering them. There is a serious disadvantage of the normalization process [13. But in many real-world applications, it is not possible to restrict the membership function to the normal form, and we should avoid it. To this end, a method - we call approximation method- with representation of the GTFNs is proposed to find an approximation solution close to the optimal solution for the FSFCTP.

The rest of the paper is organized as follows: in Section 2, we review some of the basic definitions and arithmetic between two the GTFNs. In section 3, formulation of the SFCTP is recalled. The FSFCTP is presented in section 4.

In the next section, we develop and proposed the approximation method to finding fuzzy initial solution for the FSFCTP. To illustrate the proposed methods we solve a numerical example, in section 6. Finally conclusions are discussed in section 7.

Preliminaries

In this section, we briefly review some fundamental definitions and basic notation of the fuzzy set theory in which will be used in this paper.

Definition 1 [13]: If X is a collection of objects denoted generically by x, then a fuzzy set in X is a set of ordered pairs, $\tilde{A} = \{(x, \tilde{A}(x)) \mid x \in X\}$, where $\tilde{A}(x)$ is called the membership function which associates with each $x \in X$ a number in [0, 1] indicating to what degree x is a number.

Definition 2 [13]: A fuzzy set $\tilde{A}$ on $\mathbf{R}$ is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous function.

(b) There exist three intervals [a, b], [b, c] and [c, d] such that $\tilde{A}$ is strictly increasing on [a, b], equal to 1 on [b, c], strictly decreasing on [c, d] and equal to 0 elsewhere.

Definition 3 [13]: A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number (TFN) if its membership function is given by

$$\tilde{A}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x < b \\ 1, & b \le x \le c \\ \dfrac{x-d}{c-d}, & c < x \le d \\ 0, & otherwise \end{cases}$$

Definition 4 [6]: A fuzzy set $\tilde{A}$, defined on $\mathbf{R}$, is said to be generalized fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous function.

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(b) There exist two intervals [a, b] and [c, d] such that is strictly increasing on [a, b] and strictly decreasing on [c, d].

(c) $\tilde{A}(x) = w$ for all $x \in [a, b]$, where $0 \le w < 1$.

Definition 5 [6]: A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number (GTFN) if its membership function is given by

$$\tilde{A}(x) = \begin{cases} w\dfrac{x-a}{b-a}, & a \le x < b \\ w, & b \le x \le c \\ w\dfrac{x-d}{c-d}, & c < x \le d \\ 0, & otherwise \end{cases}$$

If w = 1, then the GTFN $\tilde{A} = (a, b, c, d; w)$ is called a TFN and denoted as $\tilde{A} = (a, b, c)$.

In this subsection, we reviewed arithmetic operations on GTFNs are taken from [10].

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two GTFNs. Define,

$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; w)$, where $w = \min\{w_1, w_2\}$.

$\tilde{A}!\ \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; w)$, where $w = \min\{w_1, w_2\}$.

$$\theta\tilde{A} = \begin{cases} (\theta a_1, \theta b_1, \theta c_1, \theta d_1; w_1) & \theta > 0, \\ (\theta d_1, \theta c_1, \theta b_1, \theta a_1; w_1) & \theta < 0, \end{cases}$$

A ranking function is suited to compare the fuzzy numbers. A ranking function is defined as, $R: F(\mathbf{R}) \to \mathbf{R}$, where $F(\mathbf{R})$ is a set of fuzzy numbers, that is, a mapping which maps each fuzzy number into the real line. Now, suppose that $\tilde{A}$ and $\tilde{B}$ be two GTFNs.

Therefore,

1. $R(\tilde{A}) > R(\tilde{B})$ iff $\tilde{A} > \tilde{B}$ i.e., minimum $\{\tilde{A}, \tilde{B}\} = \tilde{B}$.
2. $R(\tilde{A}) < R(\tilde{B})$ iff $\tilde{A} < \tilde{B}$ i.e., minimum $\{\tilde{A}, \tilde{B}\} = \tilde{A}$.
3. $R(\tilde{A}) = R(\tilde{B})$ iff $\tilde{A} \approx \tilde{B}$ i.e., inimum $\{\tilde{A}, \tilde{B}\} = \tilde{A} = \tilde{B}$.

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ be any GTFN,

Then

$$R(\tilde{A}) = w_1\left(\frac{a_1 + b_1 + c_1 + d_1}{4}\right).$$

Now, let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two GTFNs, then to compare $\tilde{A}$ and $\tilde{B}$, we use the following steps [15]:

1. Find w = minimum (w_1, w_2).
2. Find $R(\tilde{A}) = w\left(\dfrac{a_1 + b_1 + c_1 + d_1}{4}\right)$, and $R(\tilde{B}) = w\left(\dfrac{a_2 + b_2 + c_2 + d_2}{4}\right)$.
3. i) If $R(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} > \tilde{B}$,
   ii) If $R(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} < \tilde{B}$,
   iii) If $R(\tilde{A}) = R(\tilde{B})$ then $\tilde{A} \approx \tilde{B}$.

Step fixed charge transportation problem

Consider a TP with *m* sources and *n* destinations. Each of the source i=1,2,…,m has S_i units of supply, and each destination j=1,2,…,n has a demand of D_j units and also, each of the *m* source can ship to any of the *n* destinations at a shipping cost per unit c_{ij} plus a fixed cost f_{ij} assumed for opening this route (i,j). Let x_{ij} denote the number of units to be shipped from source i to destination j . We need to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized. Then, the mixed integer programming formulation for the SFCTP is well known [16]:

$$\text{Min} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + g_{ij}\,y_{ij})$$

s.t

$$\sum_{j=1}^{n} x_{ij} = S_i, \qquad i=1,2,\dots,m, \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} = D_j, \qquad j=1,2,\dots,n$$

$$x_{ij} \ge 0, \qquad i=1,2,\dots,m,\ j=1,2,\dots,n,$$

$$y_{ij} = \begin{cases} 1, & x_{ij} > 0, \\ 0, & otherwise \end{cases}$$

The fixed cost g_{ij} for route *(i,j)* is proportional to the transported amount through its route. This consists of a fixed cost $k_{ij,1}$ for opening the route *(i,j)* and an additional

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cost $k_{ij,2}$ when the transported units exceeds a certain amount A_{ij}. Thus, $g_{ij} = b_{ij,1}k_{ij,1} + b_{ij,2}k_{ij,2}$, where

$$b_{ij,1} = \begin{cases} 1, & x_{ij} > 0, \\ 0, & otherwise \end{cases},$$

$$b_{ij,2} = \begin{cases} 1, & x_{ij} > A_{ij}, \\ 0, & otherwise \end{cases}$$

and $k_{ij,1}, k_{ij,2}, g_{ij}, A_{ij}$ are nonnegative real numbers.

Also, we assume that $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$, that is, it be balanced, if it be unbalanced, can by introducing a dummy source or a dummy destination be converted to a balanced transportation problem.

Note that, if all $A_{ij} > \min\{S_i, D_j\}$, then the SFCTP becomes a FCTP with a single fixed cost $k_{ij,1}$. Also, in the model (1), g_{ij} have two steps. It could have multiple steps, depending on the problem structure.

Kowalski & Lev[16], in the first time, proposed two heuristic algorithms to get a good solution per converting the SFCTP to the TP and improved this solution with using single stepping-stone moves. The formulas considered by Kowalski & Lev [16] to obtain good solution were as follows:

$$C_{ij} = \frac{k_{ij,1} + k_{ij,2}}{M_{ij}} + c_{ij}, \qquad \text{(I)}$$

And

$$C_{ij} = \frac{k_{ij,2}}{M_{ij} - A_{ij}} + c_{ij}, \qquad \text{(II)}$$

Altasan et.al [1], show that the formula (II) has a few drawbacks. Also, they have proposed three formulas to obtain good feasible solution for the SFCTP.

Fuzzy step fixed charge transportation problem

Now, we assume that the transportation costs and fixed charge costs to open a route *(i,j)* denote by $\tilde{c}_{ij}, \tilde{g}_{ij}$, respectively, which are not deterministic numbers, but are the GTFN, so, total transportation costs becomes fuzzy as well. The FSFCTP is of the following mathematical form:

$$\text{Min} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}x_{ij} + \tilde{g}_{ij}\, y_{ij})$$

s.t

$$\sum_{j=1}^{n} x_{ij} = S_i, \qquad i=1,2,\ldots,m, \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = D_j, \qquad j=1,2,\ldots,n$$

$$x_{ij} \geq 0, \qquad i=1,2,\ldots,m\,,\ j=1,2,\ldots,n,$$

$$y_{ij} = \begin{cases} 1, & x_{ij} > 0, \\ 0, & otherwise \end{cases}$$

Assume that the fuzzy fixed cost to open a route *(i,j)* is $\tilde{k}_{ij,1}$, if the output is less than or equal to A_{ij}. When the shipment exceeds A_{ij}, an additional fixed cost of $\tilde{K}_{ij,2}$ units is incurred.

Thus, $\tilde{g}_{ij}$ which is the total fixed charge associated with route *(i,j)* is $\tilde{g}_{ij} = b_{ij,1}\tilde{k}_{ij,1} + b_{ij,2}\tilde{k}_{ij,2}$, where

$$b_{ij,1} = \begin{cases} 1, & x_{ij} > 0, \\ 0, & otherwise \end{cases},$$

$$b_{ij,2} = \begin{cases} 1, & x_{ij} > A_{ij}, \\ 0, & otherwise \end{cases},$$

and $\tilde{k}_{ij,1}, \tilde{k}_{ij,2}, \tilde{g}_{ij}$ are the GTFNs.

The initial solution for the fsfctp

To the best of our knowledge, till now there is no algorithm for solving the FSFCTP, so, any method which provides a good solution should be considered useful. Here, we develop the formula (I) of Kowalski & Lev [16] to fuzzy environment and propose other two formulae to obtain a good solution of the FSFCTP per converting the FSFCTP to the FTP. The proposed formulas are as follows:

i) The generalized formula of Kowalski and lev,

$$\tilde{C}_{ij} = \frac{\tilde{k}_{ij,1} \oplus \tilde{k}_{ij,2}}{\tilde{M}_{ij}} \oplus \tilde{c}_{ij}, \qquad \text{(III)}$$

ii) The proposed formula,

$$\tilde{C}_{ij} = \frac{\tilde{k}_{ij,2} \oplus \tilde{c}_{ij}(M_{ij} - A_{ij})}{M_{ij}}, \qquad \text{(IV)}$$

iii) The proposed formula,

$$\tilde{C}_{ij} = \frac{A_{ij}}{M_{ij}}\tilde{k}_{ij,1} \oplus \frac{M_{ij} - A_{ij}}{M_{ij}}(\tilde{k}_{ij,1} \oplus \tilde{k}_{ij,2}) \oplus \tilde{c}_{ij}, \qquad \text{(V)}$$

Next, to get the fuzzy optimal solution of the FTP, we utilize some of the methods available to obtain fuzzy initial solution and fuzzy optimal solution [15]. The steps to find the good solution of the FSFCTP are as follows:

Step 1. Use the formulas (III), (IV) and (V) to convert the given the FSFCTP into the FTP.

Step 2. Apply the proposed generalized fuzzy north-west corner method, or generalized fuzzy least-cost method, or generalized fuzzy Vogel's approximation method [15] to obtain fuzzy initial basic feasible solution of the FTP.

Step 3. Apply fuzzy modified distribution method [15] to obtain the fuzzy optimal solution for the FTP.

Step 4. Find the fuzzy objective value of FSFCTP by putting $\tilde{x}_{ij}$, obtained in step 3, in objective function of the FSFCTP.

Numerical example

Suppose that a company has three factories in three different cities of 1, 2 and 3. The goods of these factories are assembled and sent to major markets in three other cities.

The demand $(S_i, i=1,2,3)$ supply $(D_j, j=1,2,3)$ for the cities and the transportation cost associated with each route *(i,j)* are given by the Table 1. Let's also assume that there are the step fixed costs in this transportation problem. Namely the cost of sending no units along route *(i,j)* is zero; but any positive shipment incurs a cost proportional to the number of unites transported plus a step fixed cost where the fixed cost to open a route *(i,j)* is $\tilde{k}_{ij,1}$ if the output is less than or equal to A_{ij} and when the shipment exceeds A_{ij}, an additional fixed cost of $\tilde{k}_{ij,2}$ units is incurred.

Notice that both quantities of the transportation cost $(\tilde{c}_{ij}, i, j=1,2,3)$ and the fixed costs $(\tilde{k}_{ij,1}, \tilde{k}_{ij,2})$, $\text{i, j}=1,2,3$, are the GTFNs in this example as shown by the Table 1, and step values $A_{ij}=5$, for all *(i,j)*, is shown in Table 2. The company wants to determine the fuzzy quantity of the product that should be transported from each of the sources to each destination such that the total fuzzy transportation costs are minimized.

Table 1. The parameters of example

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	(1,4,9,19;0.5)	(1,2,5,9;0.4)	(2,5,8,18;0.5)
$S_1=20$	(8,9,12,26;0.5)	(3,5,8,12;0.2)	(7,9,13,28;0.4)
$S_3=15$	(11,12,20,27;0.5)	(0,5,10,15;0.8)	(4,5,8,11;0.6)

Table 2. The fuzzy fixed charges of example

(3,5,8,10;0.3), (4,5,7,15;0.5)	(7,9,13,20;0.3), (5,8,10,12;0.5)	(3,8,18,28;0.1), (5,14,20,30;0.3)
(2,5,9,13;0.5), (8,11,20,23;0.6)	(8,13,17,20;0.4), (6,8,12,18;0.6)	(6,18,25,40;0.2) (10,20,23,41;0.6)
(0,3,8,10;0.2), (5,7,10,14;0.8)	(5,7,18,23;0.5), (6,15,17,30;0.2)	(7,17,20,28;0.3) (4,10,15,20;0.7)

Solution: The above problem is balanced, because, $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j = 50,$ the steps to get of the initial solution are as follows.

Step1. The fuzzy coefficients matrices generated using the formulae (III), (IV), and (V) is shown in Table 3, Table 4, Table 5, respectively.

Table 3. The fuzzy coefficient matrix using formula (III)

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	(1.7,5,10.5,21.5; 0.3)	(1.8,3.13,6.53,11.1 3;0.3)	(2.8,7.2,11.8,23. 8;0.1)
$S_1=20$	(9,10.6,14.9,29. 6;0.5)	(3.7,6.05,9.45,13.9; 0.2)	(8.6,12.8,17.8,3 6.1;0.2)
$S_3=15$	(11.5,13,21.8,29 .4;0.2)	(0.73,6.47,12.33,18 .53;0.2)	(5.1,7.7,11.5,15. 8;0.3)

Table 4. The fuzzy coefficient matrix using formula (IV)

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	(0.9, 21.5, 5.2, 11;0.5)	(1, 1.87, 4, 6.8;0.4)	(1.5, 3.9, 6, 12;0.3)
$S_1=20$	(4.8, 5.6, 8, 15.3;0.5)	(2.55, 4.15, 6.6, 9.9;0.2)	(4.5, 6.5, 8.8, 18.1;0.4)
$S_3=15$	(6,6.7,11,14.9;0.5)	(0.4, 4.33, 7.8, 12;0.2)	(2.4, 3.5, 5.5, 7.5;0.6)

Table 5. The fuzzy coefficient matrix using formula (V)

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	(6,11.5,20.5,36.5;0 .3)	(11.33,16.33,24.6 7,37;0.3)	(7.5,20,36,61;0. 1)
$S_1=20$	(14,19.5,31,50.5;0. 5)	(15.5, 24, 34, 45.5;0.2)	(18,37,49.5,88.5 ;0.2)
$S_3=15$	(13.5,18.5,33,44;0. 2)	(9, 22, 39.3, 58;0.2)	(13,27,35.5,49;0 .3)

Step2. Now, we apply the generalized fuzzy north-west corner method, to obtain of the initial basic feasible solution of the FTP, obtained in Step 1. The initial basic feasible solution is shown in Table 6.

Table 6. The initial basic feasible solution for the FTP

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	10	5	
$S_1=20$		20	
$S_3=15$		5	10

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Step3. In this step, we use of the fuzzy modified distribution method to obtain fuzzy optimal solution for initial basic feasible solution, obtained in step 2. The fuzzy optimal solution of the FTPs, obtained in step 1, with initial basic feasible solution, obtained in step 2, is shown in Table 7and Table 8.

Step3. In this step, we use of the fuzzy modified distribution method to obtain fuzzy optimal solution for initial basic feasible solution, obtained in step2. The fuzzy optimal solution of the FTPs, obtained in step 1, with initial basic feasible solution, obtained in step 2, is shown in Table 7 and Table 8.

Table 7. The optimal solution for the FTP with formulas (III) and (IV)

	$D_1 = 10$	$D_2 = 30$	$D_3 = 10$
$S_1 = 15$	10	5	
$S_1 = 20$		20	
$S_3 = 15$	5		10

Table 8. The optimal solution for FTP with formula (V)

	$D_1 = 10$	$D_2 = 30$	$D_3 = 10$
$S_1 = 15$	10	5	
$S_1 = 20$		20	
$S_3 = 15$	5		10

Step4. By putting $\tilde{x}_{ij}$ in objective function of the FSFCTP, we have:

$$\tilde{Z}_{III} = (179, 324, 555, 869; 0.2),$$
$$\tilde{Z}_{IV} = (179, 324, 555, 869; 0.2),$$
$$\tilde{Z}_{V} = (210, 323, 538, 808; 0.2),$$

Where $\tilde{Z}_{III}, \tilde{Z}_{IV}$ and $\tilde{Z}_{V}$ are fuzzy objective value of the FSFCTP. The comparative of the fuzzy objective value to the FSFCTP for the numerical example using the formulae (III), (IV), and (V) are shown in Table 9.

As summarized in Table 9, the results using the proposed three formulae (III), (IV), and (V) are as follows: the fuzzy objective values of the FSFCTP by using the formulae (III) and (IV) is both equal. Further, the results using the formula (V) is better than the results obtained using the formulae (III) and (IV).

Table 9. Summary of fuzzy objective value of FSFCTP using different formula

Formula	fuzzy bjective value of the FSFCTP	Crisp objective value of the FSFCTP
(III)	$\tilde{Z}_{III} = (179, 324, 555, 869; 0.2)$	$R(\tilde{Z}_{III}) = 96.35$
(IV)	$\tilde{Z}_{IV} = (179, 324, 555, 869; 0.2)$	$R(\tilde{Z}_{IV}) = 96.35$
(V)	$\tilde{Z}_{V} = (210, 323, 538, 808; 0.2)$	$R(\tilde{Z}_{V}) = 93.95$

CONCLUSION

Transportation models have wide applications in the real word situations. Two real extension of this model are the fuzzy transportation problem and Fuzzy step fixed charge transportation problem. The current paper, first, tried to convert the fuzzy step fixed charge transportation problem into the fuzzy transportation problem by using generalized Kowalski's formula, and then, tried to propose two formulae for constructing fuzzy coefficient matrix as a basis for finding an initial solution of the FSFCTP. Also, a comparison was done between the result of new formulae and the generalized Kowalski's formula and was shown that the proposed formula is better than two other earlier formulae by illustrating a numerical example.

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REFERENCES

1. El-Sherbiny, M. M., Alternate mutation based artificial immune algorithm for step fixed charge transportation problem, Egyptian Informatics Journal, http://dx.doi.org/10.1016/j.eij.2012.05.003.
2. Kowalski, K., Lev, B., On step fixed charge transportation problem. OMEGA, 2008. 36: 913-917.
3. Balinski, M. L, Fixed cost transportation problems. Naval Research Logistic Quarterly, 1961. 8(1): 41–54.
4. Altassan, Kh., El-Sherbiny, M. M., Bokkasam S 2012. In Handling the step fixed charge transportation problem (ICDeM2012), Kedah, Malaysia, March 15–18.
5. Zadeh, L. A., Fuzzy sets. Information and Control, 1965. 8: 338–353.
6. Bellman, R., Zadeh, L, Decision making in a fuzzy environment, Management Science, 1970. 17 4: 141–164.

7. Chanas, S., Kolodziejczyk, W., Machaj, A, A fuzzy approach to the transportation problem. Fuzzy Sets and Systems, 1984. 13: 211–221.
8. Chanas, S., Kuchta, D, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and Systems, 1996. 82: 299–305.
9. Kaur, A., Kumar, A., A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers, Applied Soft Computing, 2012. 12: 1201–1213.

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