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Ranking CCR-Efficient Units with Presence the Indicator with Limited Resources

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Abstract

Data Envelopment Analysis (DEA) is a widely applied approach for measuring the relative efficiencies of a set of decision making units (DMUs), which use multiple inputs to produce multiple outputs. The ranking of efficient DMUs is one of the most interesting aspects of DEA method. The presence of the Indicator with Limited Sources (ILS) affects ranking methods. The ILS exists as fixed amount in a community and the DMUs can own it with their ability. When a DMU loses the same amount of the indicator, the rest of the DMUs find the ability to own some without even changing their capacities of other indicators and or vice versa. If a DMU want has more of the same amount of the indicator, the rest of the DMUs must supply it without even changing their capacity of other indicators. This paper develops a ranking method based on two famous ranking methods in DEA, AP and L_1, for the efficient DMUs, when there is variation either in an input or output ILS.

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INTRODUCTION

Data envelopment analysis (DEA) proposed by Charnes et.al [1], and further extended by others, is used to evaluate the relative efficiency of decision making units (DMUs). Jahanshahloo et al. [2] was the first one who introduced the idea of *indicator with limited sources* (ILS). There are cases where the availability of some of the input or output indicators are limited and the DMUs can own them with their ability called as ILS. A good example of ILS is where we try to measure the relative efficiency of the banks located in small community. Obviously, the funding to be invested in banks is limited and there is a competition among different branches to attract new customers. The number of people who medical care is small community is another example of ILS since it is limited and hospitals can manage to offer better healing expenses to absorb more customers. In these examples, if a DMU losses an amount of the output, the rest of the DMUs get more market share while means an increase on their output. There are literally many evidences to believe that we have limited resources among various DMUs. Total amount of budget in a system is limited and it needs to be allocated among various branches based on their relative efficiencies. Another point is that if a DMU wants more of the same input, this amount must be supported by decreasing the input in the other DMUs. In this paper, we present an improved ILS ranking method for evaluating a set of efficient DMUs against variation in an input or output ILS based on AP and l_1 methods.

This paper is organized as follows. We first review the related literature review on DEA method in section 2. Section 3 present a ranking method based on sensitivity analysis of the implementation of ILS for efficient DMUs. The proposed method of this paper is supported with an application, for ranking the branches of a famous Iranian bank in section 4. Finally conclusion remarks are given at the end to summarize the contribution of the paper.

MATERIALS AND METHODS

Data envelopment analysis

Consider n DMUs, where the j-th DMU uses input vector $X_j^T = (x_{1j}, \dots, x_{mj}) \in \mathbb{R}_+^m$ produce output vector $Y_j^T = (y_{1j}, \dots, y_{sj}) \in \mathbb{R}_+^s$, where $j \in \{1, \dots, n\}$.

In DEA literature, we normally build a production technology, called Production Possibility Set (PPS), from the observed input-output vectors of the DMUs in the sample. An input-output vector (X, Y) model locates in PPS when the output vector Y can be produced by the input

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vector X. To create the PPS, we make the following general assumptions:

(A1) All actually observed input-output combinations (X_j, Y_j), $j = 1, \dots, n$, are in PPS.

(A2) The PPS is convex set, that is, if $(\bar{X}, \bar{Y})$ and $(\ddot{X}, \ddot{Y})$ are in PPS then for any $0 \le \lambda \le 1$, (X_λ, Y_λ) is also in PPS, where $X_\lambda = \lambda \bar{X} + (1 - \lambda)\ddot{X}$ and $X_\lambda = \lambda \bar{Y} + (1 - \lambda)\ddot{Y}$.

(A3) Inputs are freely disposable, that is, if $(\bar{X}, \bar{Y})$) is in PPS then for any $X \ge \bar{X}$, $(X, \bar{Y})$ is also in PPS.

(A4) Outputs are freely disposable, that is, if $(\bar{X}, \bar{Y})$) is in PPS then for any $Y \le \bar{Y}$, $(\bar{X}, Y)$ is also in PPS.

We additionally assume that Constant Returns to Scale (CRS) holds.

(A5) If $(\bar{X}, \bar{Y})$ is in PPS then for any $0 \le \lambda$, $(\lambda \bar{X}, \lambda \bar{Y})$ is also in PPS.

On the basis of the observed input-output quantities and under the five assumptions, the PPS can be defined as follows:

$$T_C = \left\{ (X, Y) \mid X \ge \sum_{j=1}^{n} \lambda_j X_j, Y \le \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \ge 0, j = 1, \dots, n \right\}. \quad (1)$$

Here the subscript C indicates that the technology is characterized by CRS. The input-oriented linear programming problem formulation for the Charnes, Cooper, and Rhodes (CCR) [1] model for evaluation of DMU_o, $j \in \{1, \dots, n\}$, (the multiplier side) is as follows:

$$\begin{aligned} \theta_o^* = \max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, & j = 1, \dots, n, \quad (2) \\ & v_i \ge \varepsilon, & i = 1, \dots, m, \\ & u_r \ge \varepsilon, & r = 1, \dots, s \end{aligned}$$

where ε is a non-Archimedean infinitesimal.

It can be proven that $0 < \theta_o^* \le 1$ and DMU_o is efficient in the CCR model if $\theta_o^* = 1$. Otherwise, the DMU_o is inefficient (1978). Therefore, from (1), DMU_o is efficient, if there are $v_i \ge \varepsilon$, $i = 1, \dots, m$ and $u_r \ge \varepsilon$, $r = 1, \dots, s$, such that

$$\begin{aligned} & \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} = 0, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, & j = 1, \dots, n, \quad (3) \end{aligned}$$

Without loss of generality it can see that $\sum_{r=1}^{s} u_r - \sum_{i=1}^{m} v_i = 1$, because if v_i and u_r satisfy in (3), then $\bar{v}_i = t v_i$ and $\bar{u}_i = t u_i$ also satisfy, where

$$t = \frac{1}{\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i}.$$

Ranking

In this section, a method for ranking efficient units will be presented that involve all indexes, including ILS indices. This is a method based super efficiency such that for ranking efficient units, it initially removes target unit from PPS, so defines a measure distance from the unit under evaluation to the frontier of new PPS. Naturally, the efficient unit that has more space to the boundary has a higher rank order, because this unit can has a more influence in PPS and is more elusive than others. Clearly, the unit that can generate a bigger PPS has a greater value then has a higher rank order. The criterion used in this way to measure the distance is L_1 norm that for the first time by Jahanshahloo et al. [3] was used.

Suppose that DMU_o is an efficient DMU and the k-th output has a limited source. The changing amount of y_{kj}, $j = 1, \dots, n$, is denoted by α_j, such that $0 \le \alpha_o \le y_{ko}$ and $0 \le \alpha_j \le p_j$, $j \ne o$, where p_j is the almost increasing y_{kj} such that DMU_j is able to produce $Y_j' = (y_{1j}, \dots, y_{kj}', \dots, y_{sj})$ by the same inputs X_j, where

$$\begin{aligned} & y_{ko}' = y_{ko} - \alpha_o \\ & y_{kj}' = y_{kj} + \alpha_j, \quad j \ne o \qquad (4) \end{aligned}$$

and $\sum_{j \ne o} \alpha_j \le \alpha_o \cdot p_j \ge y_{ko}$ means that α_j can be increased without any limitation.

Suppose that the attraction contribution of α_o by DMU_j is specified and is denoted by w_j where

$$W = (w_1, \dots, w_n), \quad w_o = 0, w_j \ge 0, \sum_{j=1}^{n} w_j \le 1$$

Therefore, we have $\alpha_j = w_j \alpha_o$. The stability interval of DMU_o is as $[0, \alpha_o(W)]$, where $\alpha_o(W)$ has the most amount α_o such that DMU_o with input-output vector (X_o, Y_o') holds over as an efficient DMU among other DMUs with input-output vector (X_j, Y_j'). From (3), we have

$$\begin{aligned} \alpha_o(W) = \max \quad & \alpha_o \\ s.t. \quad & \sum_{r \ne k} u_r y_{ro} - u_k \alpha_o - \sum_{i=1}^{m} v_i x_{io} = 0, \\ & \sum_{r \ne k} u_r y_{rj} + u_k w_j \alpha_o - \sum_{i=1}^{m} v_i x_{ij} \le 0, & j \ne o, \\ & \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \\ & 0 \le \alpha_o \le y_{ko}, \\ & w_j \alpha_o \le p_j, & j \ne o, \\ & v_i \ge \varepsilon, & i = 1, \dots, m, \\ & u_r \ge \varepsilon, & r = 1, \dots, s. \end{aligned}$$

The above model is a nonlinear programming problem which transforms to linear form by $\bar{\alpha}_o = u_k \alpha_o$.

Now, suppose that the contribution of each DMU of α_o is unknown. Hence, $\alpha_o(W)$ is a function of W. Let α_o^L be the minimum upper bound interval stability of $\alpha_o(W)$. That is

$$\alpha_o^L = \min\{\alpha_o(W) \mid W = (w_1, \dots, w_n), w_o = 0, w_j \ge 0, \textstyle\sum_{j=1}^{n} w_j \le 1\}. \quad (7)$$

The amount of α_o^L is obtained by solving a $Min\,Max$ nonlinear programming problem. Jahanshahloo et al. [4] provided a method for calculating α_o^L. It is obtained by solving $n - 1$ linear programming problems as follows

$$\alpha_o^L = z^q = \min\{z^l \mid l = 1, \dots, n, l \ne o\} \qquad (8)$$

that

$$z^l = \frac{1}{u_k^*}\min \quad z - \varepsilon\left(\sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_i^-\right) \qquad l = 1,\dots,n, l \neq o$$

$$\begin{aligned}
s.t. \quad & \sum_{j=1}^{n} y_{1j}\lambda_j + z - y_{ko}\,\beta - \sum_{j\neq o} p_j\,\mu_j - s_k^+ = 0, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j + z - s_r^+ = 0, && r \neq k, \\
& \sum_{j=1}^{n} x_{ij}\lambda_j - z + s_i^- = 0, && i = 1,\dots,m \\
& -\lambda_o + \beta + \lambda_l + \mu_l \geq 1, \\
& \lambda_j \geq 0, && j \neq o, \\
& \mu_j \geq 0, && j \neq o, \\
& \beta \geq 0, \\
& z, \lambda_o \quad free
\end{aligned} \tag{9}$$

where u_k^* is optimal solution $u_k$in (6) for W^* such that W^* is optimal solution of (7) corresponding to α_o^L. With reference to the definitions of the concepts of DEA, PPS with constant returns to scale is obtained as follows

$$T_c = \left\{ (X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j \;, 0 \leq Y \leq \sum_{j=1}^{n} \lambda_j Y_j \;, \lambda_j \geq 0 \;, j = 1,\dots,n \right\}$$

Where $X_j = (x_{1j},\dots,x_{mj}) \geq 0$ and $Y_j = (y_{1j},\dots,y_{sj}) \geq 0$ are nonzero vectors corresponding to the values of input and output of $DMU_j\ (j = 1,\dots,n)$, respectively.

As is customary in super efficiency oriented method, for evaluating of the rank order of target efficient unit DMU_o, we initially remove it from T_c. With its departure from the PPS, amount of its k-th output provide for rest DMUs and this changes can influences the PPS. α_o is an amount of k-th output of DMU_othat released to is provided an opportunity to rest units improve their k-th output. So with regard to Y_j' define in (4) and input-output vector (X_j, Y_j') instead of (X_j, Y_j) for $DMU_j\ (j \neq o)$, the PPS $T_c'(\alpha_1,\dots,\alpha_o,\dots,\alpha_n)$ is obtained as follows which is related to $\alpha_o$value and its distribution among other DMUs namely, $\alpha_j\ (j \neq o)$

$$T_c'(\alpha_1,\dots,\alpha_o,\dots,\alpha_n) = \left\{ (X,Y) | X \geq \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j X_j \;, 0 \leq Y \leq \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j Y_j' \;, \lambda_j \geq 0 \;, j = 1,\dots,n \;, j \neq o \right\}$$

and in component form

$$T_c'(\alpha_1,\dots,\alpha_o,\dots,\alpha_n) = \{(x_1,\dots,x_m, y_1,\dots,y_s) | \; x_i \geq \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \;, i = 1,\dots,m \;,$$

$$0 \leq y_r \leq \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \;, r = 1,\dots,s \;, r \neq k \;, 0 \leq y_k \leq \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j (y_{kj} + \alpha_j) \;,$$

$$\lambda_j \geq 0 \;, j = 1,\dots,n \;, j \neq o\} \tag{10}$$

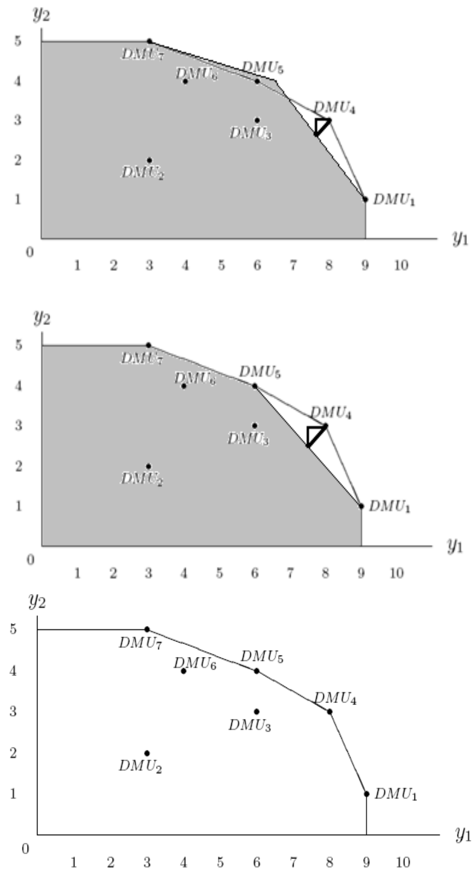
Where $\alpha_1,\dots,\alpha_o,\dots,\alpha_n$ satisfy in the assumptions discussed.

Note that new PPS is not necessarily subsetT_c. Combine different of α_j can different sets created. For example, Table 1 shows the summary input and output data of 7 DMUs that DMU_j uses a single inputx_{1j}, whose value is normalized to 1, to produce two outputs (y_{1j}, y_{2j}) such that their first output is ILS type. The PPS in outputs space is portrayed in part (a) of Fig. 1.

Table 1. Data of seven DMUs

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7
x_{1j}	1	1	1	1	1	1	1
y_{1j}	9	3	6	8	6	4	3
y_{2j}	1	2	3	3	4	4	5

DMU_4 is an efficient unit(because located on the efficiency frontier, for more information, see [5]). With the removal it of PPS, Farell frontier (is part of the efficiency frontier that is obtained from its confluence with plane $x = 1$, see [5]) can be different to various combination of α_js. New PPS is marked as full color in Fig 1. Part (b) shows the case that all α_o amount allocated to units is inefficient such that this change has not caused the efficiency frontier improved in the absence of DMU_4. But in part (c), the case is shown that some of α_o is dedicated to efficient unit DMU_5 such that has caused new efficiency frontier improved somewhat in the absence of DMU_4.



(a)

(b)

(c)

Fig. 1. The Production Possibility Set (PPS) in outputs space

The PPS obtained from deleting a target efficient unit, with the release of the α_o value of k-th its output, is denoted by $T_c'(\alpha_o)$ which includes all combinations of $\alpha_o$distribution. That is

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$$T_c'(\alpha_o) = \{(x_1, \dots, x_m, y_1, \dots, y_s) \mid x_i \geq \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij}, i = 1, \dots, m,$$

$$0 \leq y_r \leq \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj}, r = 1, \dots, s, r \neq k,$$

$$0 \leq y_k \leq \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j (y_{kj} + \alpha_j), \sum_{j \neq o} \alpha_j = \alpha_o,$$

$$0 \leq \alpha_j \leq p_j, j = 1, \dots, n, j \neq o,$$

$$\lambda_j \geq 0, j = 1, \dots, n, j \neq o\}. \quad (11)$$

Obviously, for any combination of such $(\alpha_1, \dots, \alpha_o, \dots, \alpha_n)$ with

$$\sum_{j \neq o} \alpha_j \leq \alpha_o, \qquad 0 \leq \alpha_j \leq p_j, j = 1, \dots, n, j \neq o$$

we have

$$T_c'(\alpha_1, \dots, \alpha_o, \dots, \alpha_n) \subseteq T_c'(\alpha_o)$$

Including when the overall α_o will be allocated to individual units (Condition in terms of the allocation applies)

$$T_c'(\alpha_o, 0, \dots, 0, \alpha_o, 0, \dots, 0) \subseteq T_c'(\alpha_o)$$

$$T_c'(0, \alpha_o, 0, \dots, 0, \alpha_o, 0, \dots, 0) \subseteq T_c'(\alpha_o)$$

$$\dots$$

$$T_c'(0, 0, \dots, 0, \alpha_o, 0, \dots, \alpha_o) \subseteq T_c'(\alpha_o).$$

Actually, $T_c'(\alpha_o)$ is a collection that includes input-output vectors as (X, Y) such that is obtained by assumptions A1-A5 whereas it uses the vectors (X_j, Y_j') instead of corresponding observed vectors (X_j, Y_j), $j \neq o$, such that $Y_j' = Y_j + \bar{\alpha}_j$ and $\bar{\alpha}_j = \min\{p_j, \alpha_o^L\}$.

Therefore

$$T_c'(\alpha_o) = \{(x_1, \dots, x_m, y_1, \dots, y_s) \mid x_i \geq \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij}, i = 1, \dots, m,$$

$$0 \leq y_r \leq \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj}, r = 1, \dots, s, r \neq k,$$

$$0 \leq y_k \leq \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j (y_{kj} + \bar{\alpha}_j)$$

$$\lambda_j \geq 0, j = 1, \dots, n, j \neq o\}. \quad (12)$$

Note here it is each vector in $T_c'(\alpha_o) - T_c$ occur then be prevented for happening other points of this set. Hence although there is a possible for accruing all vectors of $T_c'(\alpha_o) - T_c$ but this depends to other vectors in set do not occur. This issue may cause the image vector of target DMU, (X_o, Y_o'), is not located on the efficiency frontier, therefore cannot be efficiently in $T_c'(\alpha_o)$.

In a graphical display of this issue, consider the previous example, again. Fig 2 shows the set $T_c'(\alpha_4)$, obtained from the removal DMU_4.

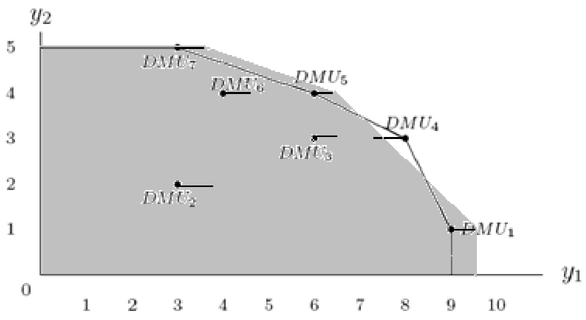

Fig. 2. The set $T_c'(\alpha_4)$ in outputs space

In Fig 2, $T_c'(\alpha_4)$ is portrayed as color area. In this case, minimal loss in the first output of DMU_4, $\bar{\alpha}_4$, is an amount that (X_4, Y_4') is located on new efficiency frontier for a distribution of it. But when this amount will be allocated to all rest DMUs, then the vector (X_4, Y_4') is not located on the efficiency frontier while we are looking for the point image of DMU_4 that is located on the efficiency frontier.

Hence the distance (X_o, Y_o) to (X_o, Y_o') may does not represent the distance of (X_o, Y_o) to $T_c'(\alpha_o)$. Suppose the point $(\bar{X}, \bar{Y}) = (\bar{x}_1, \dots, \bar{x}_m, \bar{y}_1, \dots, \bar{y}_s)$ of $T_c'(\alpha_o^L)$ has the minimal distance from (X_o, Y_o) and consider this point as the image of (X_o, Y_o) on the frontier of $T_c'(\alpha_o^L)$, such that

$$\bar{x}_i = x_{io} + s_i^-, \quad i = 1, \dots, m,$$

$$\bar{y}_r = y_{ro} - s_r^+, \quad r = 1, \dots, s.$$

The minimal distance of $T_c'(\alpha_o^L)$ from (X_o, Y_o), that is denoted by R_o, can be consider as a criterion of the influence of (X_o, Y_o) to PPS and use it for efficient ranking as is customary in super efficiency oriented ranking methods. R_o value is obtained by using following model

$$R_o = \min \quad \Gamma_c^o(X, Y)$$

$$s.t. \quad (X, Y) \in T_c'(\alpha_o^L)$$

Where $\Gamma_c^o(X, Y)$ is the distance of (X, Y) from (X_o, Y_o). For computing of $\Gamma_c^o(X, Y)$, we can use any norm. Since by l_1 norm, we can easily transform (13) to a linear programming form, here it is used as follows

$$\Gamma_c^o(X, Y) = \| (X, Y), (X_o, Y_o) \|_1 = \sum_{i=1}^{m} |x_i - x_{io}| + \sum_{r=1}^{s} |y_r - y_{ro}|.$$

There fore

$$R_o = \Gamma_c^o(\bar{X}, \bar{Y}) = \min \sum_{i=1}^{m} |x_i - x_{io}| + \sum_{r=1}^{s} |y_r - y_{ro}|$$

$$s.t. \quad (X, Y) \in T_c'(\alpha_o^L). \quad (15)$$

The above model is a nonlinear programming problem. In order to transform it into a linear programming model, similarly Jahanshahloo et al. [4], we use the following set

$$T_c''(\alpha_o^L) = T_c'(\alpha_o^L) \cap \{X \geq X_o, Y \leq Y_o\} \quad (16)$$

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Fig. 3 is depicted set $T_c''(\alpha_o^L)$ related to the part c of Fig1 as color area.

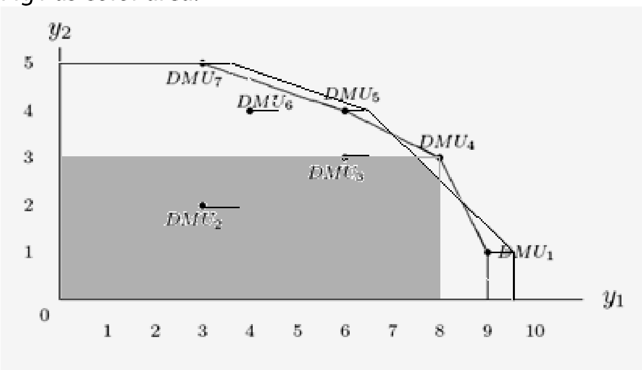

Fig. 3. The set $T_c''(\alpha_o^L)$ in outputs space

The following theorem proves that the point $(\bar{X},\bar{Y})$ belongs to $T_c''(\alpha_o^L)$.

Theorem 1. Suppose $(X_o,Y_o) \in T_c$ and is an efficient point. For every point as $(\hat{X},\hat{Y}) \in T_c'(\alpha_o^L) \setminus T_c''(\alpha_o^L)$ there is a point of of $T_c''(\alpha_o^L)$ as $(\bar{X},\bar{Y})$ such that $\Gamma_c^o(\bar{X},\bar{Y}) < \Gamma_c^o(\hat{X},\hat{Y})$.

Proof. Suppose $(\hat{X},\hat{Y}) = (\hat{x}_1,\dots,\hat{x}_m,\hat{y}_1,\dots,\hat{y}_s)$ belongs to $T_c'(\alpha_o^L) \setminus T_c''(\alpha_o^L)$ and sets $M \subseteq \{1,\dots,m\}$ and $N \subseteq \{1,\dots,s\}$ are defined as follows

$$i \in M \iff \hat{x}_i < x_{io}$$
$$r \in N \iff \hat{y}_r > y_{ro}\,.$$

Therefore $i \notin M$ if and only if $\hat{x}_i \geq x_{io}$ and also $r \notin N$ if and only if $\hat{y}_r \leq y_{ro}$. Now, we define the point $(\bar{X},\bar{Y}) = (\bar{x}_1,\dots,\bar{x}_m,\bar{y}_1,\dots,\bar{y}_s)$ as follows

$$\bar{x}_i = x_{io}, \quad i \in M$$
$$\bar{x}_i = \hat{x}_i, \quad i \notin M$$
$$\bar{y}_r = y_{ro}, \quad r \in N$$
$$\bar{y}_r = \hat{y}_r, \quad r \notin N\,.$$

From the definition of $(\bar{X},\bar{Y})$, we result that $\bar{X} \geq \hat{X}$ and $\bar{Y} \leq \hat{Y}$. According to A3, A4 and the definition of $T_c'(\alpha_o^L)$, we have $(\bar{X},\bar{Y}) \in T_c'(\alpha_o^L)$ and since $\bar{X} \geq X_o$ and $\bar{Y} \leq Y_o$, we can result that $(\bar{X},\bar{Y}) \in T_c''(\alpha_o^L)$. Now with regard to subject discussed, we have

$$\Gamma_c^o(\hat{X},\hat{Y}) = \sum_{i\in M}|x_i - x_{io}| + \sum_{i\notin M}|x_i - x_{io}| + \sum_{r\in N}|y_r - y_{ro}| + \sum_{r\notin N}|y_r - y_{ro}|$$
$$> \sum_{i\notin M}|x_i - x_{io}| + \sum_{r\notin N}|y_r - y_{ro}| = \Gamma_c^o(\bar{X},\bar{Y})\,. \blacksquare$$

Therefore if $R_o = \min \ \Gamma_c^o(\bar{X},\bar{Y})$ in model (15), then $(\bar{X},\bar{Y}) \in T_c''(\alpha_o)$. Hence in order to transform model (15) into a linear programming problem, without the optimal solution of (15) is changed, we add it two following constraints $X \geq X_o;\ 0 \leq Y \leq Y_o$.

With the above interpretations, R_o value is obtained from the following problem

$$\begin{aligned} R_o = \ & \Gamma_c^o(\bar{X},\bar{Y}) = \min \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \beta \\ s.t.\ & (X,Y) \in T_c'(\alpha_o^L) \\ & X \geq X_o \\ & Y \leq Y_o \end{aligned} \qquad (17)$$

where $\beta = -\sum_{i=1}^{m} x_{io} + \sum_{r=1}^{s} y_{ro}$ and is a constant. Finally the criterion of efficient units ranking for target DMU is obtained as follows

$$\begin{aligned} R_o = \ & \Gamma_c^o(\bar{X},\bar{Y}) = \min \ \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \beta \\ s.t.\ & x_i \geq \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij}, && i = 1,\dots,m, \\ & 0 \leq y_r \leq \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj}, && r = 1,\dots,s, r \neq k, \\ & 0 \leq y_k \leq \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j (y_{kj} + \bar{\alpha}_j), \\ & x_i \geq x_{io}, && i = 1,\dots,m, \\ & y_r \leq y_{ro}, && r = 1,\dots,s, \\ & \lambda_j \geq 0, && j = 1,\dots,n, j \neq o \end{aligned} \qquad (18)$$

where $\bar{\alpha}_j = \min\{p_j, \alpha_o^L\}$. Therefore, the efficient unit that has more R_o value, has a higher rank order.

Consider previous example again. Table 2 shows the amounts of α_o^L from (8), R_o from (18), the image vectors $(\bar{x}_1,\bar{y}_1,\bar{y}_2)$ and the rank orders of each efficient DMUs.

Table 2. The results of efficient DMUs ranking

	DMU_1	DMU_4	DMU_5	DMU_7
$(\bar{x}_1,\bar{y}_1,\bar{y}_2)$	(1,9,1)	(1,8,3)	(1,6,4)	(1,3,5)
α_o^L	0.4996	0.5998	0.3340	2.9945
R_o	0.5004	0.4002	0.0664	1.0000
$(\bar{x}_1,\bar{y}_1,\bar{y}_2)$	(1,8.4996,1.0000)	(1,8.0000,2.5998)	(1,6.0000,3.9336)	(1,3.0000,40000.)
Step	2	3	4	1

From the results, we have that although all efficient units have a positive value for the minimum upper bound interval stability (α_o^L) but DMU_1 can only loss some of its first output and still remain as an efficient DMU.

If the ranking were done solely based on ILS output or α_o^L, then was moved instead of DMU_1 and DMU_7 in the ranking while this does not happened when all the indicators involved.

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CONCLUSION

This paper has provided a method for ranking efficient units that have an ILS indicator. As has been the definition of ILS indicators, remove the data of an observed DMU from the input-output observed vectors set causes the rest DMUs available to be some of the ILS indicator. This makes the ranking methods in DEA to change which here, one of the most complete ranking methods in DEA was generalized based on two famous methods, AP and l_1, for the efficient DMUs, when there is variation either in an input or output ILS. In continuing this study, researchers interested can examine the case that DMUs include several ILS indicators.

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