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Proposing a Mathematical Model for Layout in a Cellular Manufacturing System under Dynamic Conditions

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Abstract

The fundamental function of Cellular manufacturing system (CMS) is based on definition and recognition of type of similarity among the parts that must be produced in a planning period. Cell formation (CF) and cell machine layout design are two major steps in implementation of CMS. So far there have been numerous researches concerning to examine various types of CMS and it can be said that existing models in this area mainly focus on CF and cell machine layout design and determining cell layout, except few studies, have not been considered so much. This paper represents a multiple objective model for CF and layout considering intra-cell layout and inter-cell layout, under the condition of existence of alternative process routings for the parts, and using simultaneous solution, set of Pareto answers has been proposed. In order to solve the model, Multi Objective Scatter Search (MOSS) has been proposed and its results have been compared with the obtained results of NSGA-II algorithm. Results comparison through considered criteria showed that generally function of MOSS is better.

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INTRODUCTION

In today's world, due to the enhancements in the customer's power of choice and extension of competitive markets, organizations are required to reform their structures.

Group Technology (GT) is a production philosophy that aims to determine, categorize and assign parts to groups and part families and also it assigns machines to cells to produce these part families. This process is based on parts' characteristics and their similarity which is called Similarity Coefficient (SC).

CMS, which is the most important application of GT, overcomes the inefficiency of traditional approaches through reduction in transportation time and distance. Flow shop layout has high efficiency in mass production system, while job shop is a very flexible system for producing various parts. In fact, each of these systems does not have other benefits. CMS is an approach between these two manufacturing systems that aims to improve flexibility and efficiency to produce manufacturing groups in different sizes (see Exhibits 1, 2 & 3). In CMS, machines and parts assignment to cells must impose minimum cost to the system. After determining the assignment to machines and parts, machines locations must be determined. This issue is referred as Cell Layout (CL).

Kannan and Ghosh [3] have examined DCMS in terms of scheduling and they have illustrated that simpler scheduling can be achieved thorough DCMS. They have examined the impacts of DCMS on preparation time, flow time and work-in-process. Balakrishnan and Cheng [4] proposed a flexible two-step method for solving the cell formation problem with respect to demand variations

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using dynamic programming and machine assignment. The first phase, contains assigning machines to each period and the second phase of the method is employing dynamic programming for designing within planning periods. Rheault et al. [5] proposed the idea of dynamic cell manufacturing to overcome the flexibility reduction in designing CMS. They have considered demand and product mix in each period as a constant and certain value. Their objective function is minimizing total inter-cell relocation costs and cell restructuring with constraints of capacity of cell space and common Quadratic Assignment Problem (QAP) constraints. Kioon et al. [6] proposed a new mathematical model for designing DCMS. This model, in addition to considering inter-cell relocation, cell restructuring and cell capacities, considers assumptions such as level of inventory in each period, intra-cell relocation, maintenance costs and subcontracting. Schaller [7] proposed a mathematical model for CF problem that considers demand variations within the periods. Wicks and Reasor [8] proposed a mathematical model for designing manufacturing dynamic cell system based on part family and machine grouping. The objective function is minimizing total inter-cell relocation costs, fix costs of machine purchasing and cell restructuring costs with constraints of machine capacity and lower bound of cells capacity. Chen [9] presented the DCMS model to minimize material relocation costs, cell restructure and fix machine cost with constraints of lower and upper bounds of cells capacity and constant number of cells. Mungwattana [10] have examined the cell formation problem in his thesis. In his study assumptions such as multiple operational paths, batch movements of the parts, variable production costs, machine time capacity constraints, lower and upper bounds for cells capacity, number of cell constraint, demand variations, machine purchasing costs, cell restructuring cost, inter-cell relocating cost and multitask machines are considered. Safaei and Tavakkoli-Moghaddam [11] developed a new model for solving the dynamic cell formation problem considering subcontracting with assumptions such as batch movement of the parts, inter-cell relocating, demand variations in different periods, machine time capacity, maximum capacity of cells, existence of multitask machines and parts returns by purchasers. They solved the proposed model after linearization thorough branch and bound (B&B) algorithm. Safaei et al. [12] proposed nonlinear integer mathematical model, CMS, in dynamic condition to overcome demand variations and product mix. The advantages of this model is considering inter-cell and intra-cell batch relocation, operational sequence, multiple paths and multiplicity in type of machines. The main constraints in this model are upper bound of cells capacity and time capacity of machines. They solve their model by merging Mean Field Annealing (MFA) algorithm with Simulated Annealing (SA) algorithm and called the hybrid algorithm as MFA-SA. They examined the obtained findings with SA algorithm and B&B and demonstrated that MFA-SA algorithm achieves better results. Askin et al. [13] proposed a four-step algorithm for solving CF problem considering demand variations and product mix. In the proposed model, existence of multiple operational paths is considered as assumption. Phase 1 is related to assigning the activities to specific types of machines. Phase 2 is about assigning parts activity to a specific machine. Phase 3 is related to candidate cell determination for locating the machine. Phase 4 is about cell design enhancement and completion. Safaei et al. [14] solved the integer mathematical model related to CF problem with dynamic and uncertain environment, employing fuzzy programming. In this model, demand and time capacity of machines are considered as fuzzy forms. Objective function of the proposed model is to minimize total inter-cell and intra-cell relocating costs, fix and variable costs of machines and cell restructuring costs. Tavakkoli-Moghaddam et al. [15] have proposed multi criterion linear integer model that includes information such as cell capacity constraints, inter-cell relocation, multi operational paths, machines reestablishment in planning periods, existence of several single type machines and operation sequence. The objective function for proposed model is minimizing machine purchasing costs, inter-cell relocation cost, part production cost and cell restructuring cost. They solved the proposed model by SA and demonstrated that in sufficient time, more appropriate solutions will be obtained relative to B&B algorithm. Tavakkoli-Moghaddam et al [16] solved the Mungwattana's proposed model by meta-heuristic algorithms such as SA, GA and Tabu Search (TS) and after comparing with B&B. They demonstrated that SA algorithm will generate more appropriate solutions for solving this particular model. Bajestani et al. [17] have proposed a multi criterion programming model for dynamic CF. They solved the proposed model employing Scatter Search (SS) algorithm and have demonstrated that for this problem, SS algorithm outperforms multi criterion GA. Saidi-mehrabad and Safaei [18] have developed the dynamic CF model considering number of variable cells for sequential planning periods and then solved the model by neural network in deterministic condition.

The proposed model

The proposed model with assumptions, parameters, decision variables, objective function and constraints are discussed in follow.

Model assumption

1. The number of parts for production is known. All of the part operations must be performed.
2. Only one machine is available for each type.
3. Part demands are definite and must be supplied.
4. Materials are transported in batches and batch sizes are definite.
5. A part may have variable process routings. When the order of operations is not important and/or machines are capable of performing multiple operations, parts may have more than one process routing for production. Finally, one routing is selected for the part and the part is manufactured through the selected routing.

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6. Materials handling unit cost is definite for inter-cell and intra-cell moves. Due to the importance of minimizing inter-cell movements, the inter-cell materials handling unit cost is greater than the intra-cell materials handling unit cost.

7. The number of cells is known and is a parameter of the model.

8. The sizes of machines do not affect the distances of movements.

9. The cell size is limited by the total number of candidate locations for machines in each cell. The sum of the number of candidate locations in all cells is greater than the number of machines.

$$F_1 = Min\ NoI = \sum_{k=1}^{C}\sum_{j=1}^{N}\sum_{e=1}^{E_j}\sum_{i=1}^{M} W_{jei} \,.\, Z_{je} \,.\, \frac{|Y_{jk} - X_{ik}|}{2} \tag{1}$$

$$F_2 = Min\ NoV = \sum_{k=1}^{C}\sum_{j=1}^{N}\sum_{e=1}^{E_j}\sum_{i=1}^{M} \left(Z_{je}.Y_{jk}\,.X_{ik} - W_{jei}\,.Z_{je}.Y_{jk}\,.X_{ik}\right) \tag{2}$$

$$F_3 = Min\ TC^{inter} = \sum_{j=1}^{N}\sum_{e=1}^{E_j}\sum_{t=1}^{T_{je}-1}\sum_{k=1}^{C}\sum_{\substack{k'=1 \\ k\neq k'}}^{C}\sum_{l=1}^{L}\sum_{l'=1}^{L} c^{inter}.\left[\frac{D_j}{B_j}\right].Z_{je}.\left(V_{i_{je}^{t}lk}\right).\left(V_{i_{je}^{t+1}l'k'}\right).d^{inter}_{(k,k')} \tag{3}$$

$$F_4 = Min\ TC^{intra} = \sum_{j=1}^{N}\sum_{e=1}^{E_j}\sum_{t=1}^{T_{je}-1}\sum_{k=1}^{C}\sum_{l=1}^{L}\sum_{\substack{l'=1 \\ l\neq l'}}^{L} c^{intra}.\left[\frac{D_j}{B_j}\right].Z_{je}.\left(V_{i_{je}^{t}lk}\right).\left(V_{i_{je}^{t+1}l'k}\right).d^{intra}_{(i_{je}^{t},i_{je}^{t+1})} \tag{4}$$

Subject To:

$$\sum_{e=1}^{E_j} Z_{je} = 1 \qquad \forall j = 1,2,\dots,N \tag{5}$$

$$\sum_{k=1}^{C} Y_{jk} = 1 \qquad \forall j = 1,2,\dots,N \tag{6}$$

$$\sum_{k=1}^{C} X_{ik} = 1 \qquad \forall i = 1,2,\dots,M \tag{7}$$

$$\sum_{i=1}^{M} V_{ilk} \leq 1 \qquad \forall k = 1,2,\dots,C \qquad \forall l = 1,2,\dots,L \tag{8}$$

$$\sum_{k=1}^{C}\sum_{l=1}^{L} V_{ilk} = 1 \qquad \forall i = 1,2,\dots,M \tag{9}$$

$$\sum_{i=1}^{M} V_{ilk} \geq \sum_{i=1}^{M} V_{i(l+1)k} \qquad \forall l = 1,2,\dots,L-1 \qquad \forall k = 1,2,\dots,C \tag{10}$$

$$\sum_{l=1}^{L} V_{ilk} = X_{ik} \qquad \forall i = 1,2,\dots,M \qquad \forall k = 1,2,\dots,C \tag{11}$$

$$X_{ik}, Z_{je}, Y_{jk}, V_{ilk} \in \{0,1\} \qquad \forall i, \forall j, \forall e, \forall k, \forall l \tag{12}$$

Sets:

$i, i' = \{1,2,\dots,M\}$	Index set of machines
$j = \{1,2,\dots,N\}$	Index of parts
$e = 1,2,\dots,E_j$	index of part routing
$t = 1,2,\dots,T_{je}$	Index of operation index
$k, k' = 1,2,\dots,C$	Index set of cells
$l, l' = 1,2,\dots,L$	Index of machine location

D_j	Part *j* demand
B_j	Part *j* batch size
C	Number of cells
L	Maximum number of candidate locations
E_j	Number of process routings of part *j*
T_{je}	Number of operations of part *j* under process routing *e*
c^{inter}	Inter-cell materials handling cost

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c^{intra}	Intra-cell materials handling cost
$d^{inter}_{(k,k')}$	Inter-cell distance between cells *k* and *k'*
$d^{intra}_{(i,i')}$	Intra-cell distance between machines *i* and *i'*
NoI	Number of inter-cell movements
NoV	Total number of voids inside blocks
TC^{inter}	Total cost of inter-cell moves
TC^{intra}	Total cost of intra-cell moves
W_{jei}	If part *j* needs machine *i* under process routing *e* is 1 and otherwise is 0 .

Parameters:

Decision variables

$$Z_{je} = \begin{cases} 1 & \text{If process routing } e \text{ of part } j \text{ is selected} \\ 0 & \text{Otherwise} \end{cases}$$

$$Y_{jk} = \begin{cases} 1 & \text{If part } j \text{ is allocated to cell } k \\ 0 & \text{Otherwise} \end{cases}$$

$$X_{ik} = \begin{cases} 1 & \text{If machine } i \text{ is assigned to cell } k \\ 0 & \text{Otherwise} \end{cases}$$

$$V_{ilk} = \begin{cases} 1 & \text{If machine } i \text{ is located in candidate location } l \text{ in cell } k \\ 0 & \text{Otherwise} \end{cases}$$

Mathematical formulation

With respect to input parameters and variables, the proposed nonlinear model for this problem is as follow:

The first objective function minimizes the total number of exceptional elements (ones outside the blocks in the part– machine matrix) which equals the number of inter-cell movements. The coefficient 1/2 is used since the exceptional elements are taken into account twice. The second objective function maximizes the utilization of each cell by minimizing the number of voids in each block in the part–machine matrix. Objective function (3) minimizes the total cost of inter-cell movements and the Objective function (4) minimizes the total cost of intra-cell movements of parts.

Constraint (5) represents that only one process routing is selected for each part. Constraint (6) ensures that each part is allocated to only one cell. Similarly, constraint (7) denotes that each machine is assigned to one cell. Constraint (8) ensures that only one machine can be placed in each candidate location. If the total number of candidate locations is equal to the number of machines, this inequality becomes equality. Constraint (9) represents that each machine can be placed in only one location in a cell. Constraint (10) denotes that machines must be assigned from the first candidate locations in the assigned cell. In other words, the first machine is assigned to the first candidate location in the cell, the second machine to the second candidate location, etc. Furthermore, machines must be placed immediately after previous machine. That I, no unoccupied candidate location must exist between machines. Constraint (11) is the relation of the variables and constraint (12) represents types and ranges of the variables.

Solution approaches

The integrated problem of cell formation and layout design is a multi-objective problem. Therefore, multi objective techniques are used to solve this problem. In the multi objective optimization, there is Pareto optimal set instead of a single optimal solution.

Multi-objective scatter search

Scatter search is one of the popular population based meta-heuristic algorithms proposed by Glover. The concept of this method is based on combining decision rules and surrogate constraints. Various authors have used scatter search to solve CMS design problems during recent years. This algorithm generates new solutions by combining the current solutions. The initial solutions are generated using diversification generation method and then improved by improvement method. Diverse and high-quality solutions construct the Reference Set which plays an important role in this algorithm. The solutions in the Reference Set are employed to generate new solutions after subset generation and combination procedure. New solutions undergo improvement method which results in the new pool of solutions. Trial solutions create the new Reference Set. This loop continues until termination criteria are met. The characteristics of the algorithm are described in this section.

Data set generation

Parameter	**Value**
D_j	$U(100,1000)$
B_j	$U(50,100)$
L	$> \frac{M}{C}$
M	$\left[{}^{N}/_{2}\right] + \left[\sqrt{N}\right]$
c^{inter}	0.7
c^{intra}	0.3
C_1	0.7
C_2	0.3

Sizes of the generated problems

Problem No.	C	L	M	N	$\sum_{j=1}^{N} e_j$
1	2	4	6	8	10
2	3	4	10	14	20
3	4	4	14	20	30
4	5	4	17	25	40

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Results Of MOSS for first sample problem

solution	NoI	NoV	φ_1	TC^{intra}/C^{intra}	TC^{inter}/C^{inter}	φ_2	$TC^{intra}/C^{intra} + TC^{inter}/C^{inter}$
1	5	6	5.3	218	91	129.1	309
2	3	10	5.1	270	75	133.5	345

Results Of MOSS for second sample problem

solution	NoI	NoV	φ_1	TC^{intra}/C^{intra}	TC^{inter}/C^{inter}	φ_2	$TC^{intra}/C^{intra} + TC^{inter}/C^{inter}$
1	9	17	11.4	324	314	317	638
2	9	18	11.7	384	274	307	658
3	11	15	12.2	370	255	289.5	625
4	12	14	12.6	328	255	276.9	583
5	11	21	14	358	240	275.4	598

CONCLUSION

In this paper, the integrated problem of cell formation and layout design was investigated. This problem seeks to determine part families, machine groups, and inter-cell and intra-cell layout design. At first, a comprehensive review on the subject was accomplished: most of the past researches on the cell formation and layout design problem have considered only one type of layout, inter-cell or intra-cell. Sequential approach has been the dominant approach that authors have developed to tackle the problem. This problem is a multi-objective optimization problem.

Thus, the problem has Pareto optimal solutions instead of one optimal solution. Also, it is more realistic to minimize the inter-cell costs rather than the number of inter-cell movements.

REFERENCES

1. Chan, H. & Milner, D., "Direct Clustering Algorithm for Group Formation in Cellular Manufacture", Journal of Manufacturing Systems, Vol. 1, No. 1, 1982, pp. 64–76.
2. Wemmerlov, U., Hyer, N. L., "Procedures for Part. Family/machine Group Identification Problem in Cellular Manufacturing", Journal of Operations Management, Vol. 6, No. 2, Feb. 1986, pp. 125–147.
3. Kannan, V. & Ghosh, S. "Using Dynamic Cellular Manufacturing to Simplify Scheduling in Cell Based Production Systems", Omega, Vol. 23, No. 4, 1995, pp. 443–452.
4. Balakrishnan, J. & Cheng, C. H., "Multi-Period Planning and Uncertainty Issues in Cellular Manufacturing: a Review and Future Directions", European Journal of Operational Research, Vol. 117, No. (1–16), Feb. 2007, pp. 281–309.
5. Rheault, M., Drolet, J. & Abdulnour, G., "Physically Reconfigurable Virtual Cells: A Dynamic Model for a Highly Dynamic Environment"", Computers & Industrial Engineering, Vol. 29, No. (1–4), Sep. 1995, pp. 221–225.
6. Kioon, S. A., Bulgak, A. A. & Bektas, T., "Integrated Cellular Manufacturing Systems Design with Production Planning and Dynamic System Reconfiguration". European Journal of Operational Research, Vol. 192, No. 2, Jan. 2009, pp. 414–428.
7. Schaller, J. "Designing and Redesigning Cellular Manufacturing Systems to Handle Demand Changes", Computers & Industrial Engineering, Vol. 53, 2007, pp. 478–490.
8. Wicks, E. M., & Reasor, R. J. (1999). "Designing Cellular Manufacturing Systems with Dynamic Part Populations", IIE Transactions, Vol. 31, No. 1, Jan. 1999, pp. 11–20.
9. Chen, M., (1998). "A Mathematical Programming Model for Systems Reconfiguration in a Dynamic Cell Formation Condition", Annals of Operations Research, Vol. 77, No. 1, 1998, pp. 109–128.
10. Mungwattana, A., "Design of cellular Manufacturing Systems for Dynamic and Uncertain Production Requirement with Presence of Routing Flexibility", Ph.D. thesis, 2009, Virginia Polytechnic Institute and State University, Blackburg, VA.
11. Safaei, N. & Tavakkoli_moghaddam, R., "Integrated Multi-Period Cell Formation and Subcontracting Production Planning in Dynamic Cellular Manufacturing Systems", International Journal of Production Economics, Vol. 120, No. 2, Aug. 2009, pp. 301–319.
12. Safaei, N., Saidi-Mehrabad, M. & Jabal-Ameli, M. S., "A Hybrid Simulated Annealing for Solving an Extended Model of Dynamic Cellular Manufacturing System", European Journal of Operational Research, Vol. 185, No. 2, 2008, pp. 563–592.
13. Askin, R. G., Selim, H. M. & Vakharia, A. J., "A Methodology for Designing Flexible Cellular Manufacturing Systems", IIE Transactions, Vol. 29, No. 7, 1997, pp. 599–610.
14. Safaei, N., Saidi_mehrabad, M. & Babakhani, M., "Designing cellular manufacturing systems under dynamic and uncertain conditions", J Intell Manuf, Vol. 18, 2007, pp.383–399.

15. Tavakkoli_moghaddam, R., Safaei, N. & Sassani, F., "A New Solution for a Dynamic Cell Formation Problem with Alternative Routing and Machine Costs Using Simulated Annealing", Journal of the Operational Research Society, Vol. 59, 2008, pp. 443–454.
16. Tavakkoli_moghaddam, R., Aryanezhad, M. B., Safaei, N. & Azaron, "A Solving a Dynamic Cell Formation Problem Using Metaheuristics", Applied Mathematics and Computation, Vol. 170, No. 2, Nov. 2005, pp. 761–780.
17. Bajestani, A. M., Rabbani, M., Rahimi-Vahed, A. R. & Khoshkhou, G. B., "A Multi–Objective Scatter Search for a Dynamic Cell Formation Problem", Computers & Operations Research, 36, Mar. 2009, pp. 777–794.
18. Saidi_mehrabad, M. & Safaei, N., "A New Model of Dynamic Cell Formation by a Neural Approach", International Journal of Advanced Manufacturing Technology, Vol. 33, No. (9–10), Jul. 2007, pp. 1001–1009.