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Implementation of Statistical Quality Control in Dairy Factories Using Univariate and Multivariate Control Charts

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Abstract

Statistical quality control, the use of statistical techniques is in all stages of production, processing and marketing services, so we can have quality products in accordance with the desired characteristics of the consumers and make sure customer satisfaction. One of the main tools of statistical process control is the use of control charts. When control charts are most commonly exploited the qualitative characteristics of the study was the univariate and comply normal distribution. If the simultaneously study of qualitative characteristics to be important, parameters monitoring methods for multivariate process is prepared, but this need to know more statistical methods. Many analysts prefer the simplicity of calculations of independence hypothesis quality characteristics considered and their issues in the univariate statistical environment are examined. In this paper, a univariate control charts for technical specifications dairy prepared then Hotelling multivariate control charts for variables prepared and the results are compared with each other.

Original Article

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INTRODUCTION

Statistical quality control, the use of statistical techniques is in all stages of production, processing and marketing services, so we can have quality products according to specifications and the production of consumer and make sure customer satisfaction. Quality is beyond the skills, tools, processes and production requirements. Should be noted that if the production is not due to the production process and inspection should not be on it, we have the possibility to create a lot of waste and rework and products produced outside the controlled conditions and will eventually lead to customer dissatisfaction. In the world of competition among manufacturers is very high, produce a good quality product and the customer can make more transparent the manufacturer on the market. We must also consider the costs, if production is controlled conditions and the process is under control, less waste and rework in and it would be better products and lower prices offered. You

might think that the quality system implementation costs caused by and it seems that the product quality is equal to the product with a higher price. If you look beyond the cost of scrap and rework can also see other than it. Have abandoned the traditional view and understand this fact: "Improving the quality cannot be increased by reducing the costs associated with it," the use of statistical techniques can improve the quality of our products. This will reduce waste and reduce costs and ultimately customer satisfaction wills this means increased sales and satisfaction [1]. In fact, statistical quality control, improve process capability by reducing the variability of variables [2].

Statistical process control is defined as the application of statistical techniques to control a process. SPC is concerned with quality of conformance [3]. There are a number of tools available to the quality engineer that is effective for problem solving process. The seven quality



tools are relatively simple but very powerful tools which every quality engineer should aware. The tools are: flow chart, run chart, process control chart, check sheet, Pareto diagram, cause and effect diagram, and scatter diagram

One of the main tools of statistical process control is the use of control charts. When control charts are most commonly exploited the qualitative characteristics of the study was the univariate and comply normal distribution. Otherwise, in addition to the continuing lack of change in the distribution of the test statistic, monitoring parameters such as mean and variance pardon novel process for managers and engineers, the quality is of utmost importance. If the simultaneously study of qualitative characteristics to be important, parameters monitoring methods for multivariate process is prepared. but this need to know more statistical methods. Many analysts prefer the simplicity of calculations of independence hypothesis quality characteristics considered and their issues in the univariate statistical environment are examined [4].

In many cases it is clear that this assumption is not true, but unfortunately it can be seen users have used it incorrectly and may also make a wrong decision to support it. Several researchers have developed methods correlated multivariate quality control [5].

This assumption may be violated since the autocorrelation may effect the false alarm rate and the shift detection power. Hence, traditional control charts would be effected by this violation. This problem has been studied by many authors [6-9], and The recent works on multivariate process monitoring were made by Cheng and Cheng [7], Noorossana et al. [8], Yu and Wang [9], Taleb [10] to name a few [7,8,9,10].

Multivariate charts are developed to enable joint monitoring of several related quality characteristics. In general, these multivariate charts fall under three main categories, namely, T2-type charts, MEWMA - type charts and multivariate cumulative sum (MCUSUM) - type charts among the works on T2-type charts [11].

The first method of multivariate statistical control of processes by Hotelling in 1947 with a variety of examples of sites that were bombed in World War was presented

Existing works on this chart will be briefly reviewed: Costa and Machado [13] suggested a synthetic control chart with two-stage sampling to monitor bivariate processes. Ghute and Shirke [14] developed a synthetic T2 chart for monitoring the mean vector of a process that follows a multivariate normal distribution. Their chart was shown to outperform the Hotelling's T2 chart and the T2 chart with supplementary runs rules. Adamastor R. Tôrres et al. [15] used the multivariate control charts to monitor chromatographic profile changes and captopril stability standard changes. Tuerhong and Bum Kim [16] proposed a multivariate control chart based on the Gower distance that can handle a mixture of continuous and categorical data. Zeng et al. [17] develop a reference principal component analysis model and then enabled the establishment of multivariate control charts. Hotteling T2

and DModX charts were applied to examine batch-tobatch reproducibility of 12 test batches. Faraz et al. [18] proposed control the process using multivariate T2 control charts economically designed for monitoring key variable of delivery chain.

In this paper, a univariate control charts for technical specifications dairy prepared then Hotelling multivariate control charts for variables prepared and the results are compared with each other.

T2 Hotelling multivariate control

Hotelling's T² distribution is the multivariate analogue of the univariate t distribution for the use of known standard value µ or individual observations [19].

Under the null hypothesis of the process being in control and the assumption of independent identical multivariate normality, the chart statistic follows

Hotelling's T² is: [1]
$$T^2 = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{p,mn-m-p+1}$$

After parameter estimation, a preliminary charting for Phase I samples should be run to see whether the chart is well constructed, before stepping into Phase II to monitor the future samples. The control limits are set according to the specified level of significance:

$$UCL = \frac{p(m-1)(n-1)}{mn - m - p + 1} F_{\alpha, p, mn - m - p + 1}$$
(1)

And since usually the shift in mean vector and the increase of covariance are of interest, LCL=0. The chart signals when T2 > UCL. After confirming the process is in control, then in Phase II, the Hotelling T2 becomes, with future sample mean $(\bar{x_i})$, of size n:

$$T^{2} = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{p,mn-m-p+1}$$
(2)

 $T^{2} = \frac{p(m-1)(n-1)}{mn - m - p + 1} F_{p,mn - m - p + 1}$ (2) Similar to that in Shewhart chart, although the Phase Il samples and their mean, $\bar{x_i}$, are independent, the T2 for different Phase II samples are not independent of each other's because they share the same Phase I grand mean \overline{x} and pooled covariance matrix \overline{s} . In Phase II, however, the statistic still has an F distribution:

$$T^{2} = \frac{(m+1)(m-1)}{m^{2} - mp} F_{p,m-p}$$
 (3)

But usually $\chi^2(p)$ can be used to approximate the distribution when m is large. This chi-square approximation makes more conservative control limits than the original F distribution [19].

Case study

Among the major industries that use spc can be considered essential is the production of the dairy industry. In this paper, the implementation SPC for Nekoonam milk factory located in a five-kilometer road Nahavand - Kermanshah discussed. Three products, milk, cheese and yogurt are the main products of the plant are considered. The dairy industry in material quality control at three stages during the production process and the final step is done. In order to determine the important parameters influencing and the important role of milk products in all of the graphics quality for this product have been investigated. In this study the statistics of the average control charts (\overline{X}) changes in scope (R) for the three qualitative characteristic density, fat and acidity is intended. The remainder of the sampling and analysis of the characteristics of normal observations, \overline{X} and R charts for each of them is drawn and since these three characteristics of the raw material of the plant are correlated to each other, it is necessary to control the use of multivariate control charts. Here after testing the correlation between the characteristics of the graph Hotelling T2 is used for process control.

Determine the main parameters to design the control charts

Variable selection: According to studies, the parameters of milk are considered must get approval of their product quality, parameters such as: alcohol test, acidity, density, fat, VRB, Ecoli, PC respectively. Which in turn has important explanation about each of them chose to work on control charts. Alcohol test, alcohol content in the milk tested 10 Cc valve to alcohol and should be sustained until the value is acceptable 1-10.

Acidity: This parameter is an important factor in quality like the taste of milk is effective and allowable own factory quality control department to consider it acceptable limit is between 14 and 16. If the water must be added is less than 14 and if that is more than 16 Dough will be transferred to the production line.

Fat: Fat is one of the important parameters because of the impact they can have on milk prices and to distinguish different types of milk is of great importance.

Density (density): The density in the milk quality is of utmost importance. Laboratory to determine the extent to which it has been between 29 and 33

VRB: represents the number of bacteria in milk that is mostly due to factors such as machine operator and transferred to milk.

Ecoli: This parameter must be negative as toxicity in milk if it is positive and indicates the presence of bacteria in milk and milk is rejected.

PC: shows total count of bacteria in milk (both pathogenic and Non-pathogenic). Considering the cost of measuring the three parameters of the VRB, Ecoli and P.C and lack of laboratory equipment as well as the importance of fat, acidity and density, these three parameters chosen for the study of institutional control charts are considered.

Determination of sample size and sampling

There are generally two phases in statistical process control (SPC) applications. In Phase I, a historical set of data is considered to determine the in control process performance and understand the variation in the process over time. In Phase II, actual process monitoring is performed based on the control chart constructed in Phase I [1].

The general assumption is that the data are normally and independently distributed with mean and standard deviation when the process is in control. If this assumption is violated, the control charts are effected by the violation of independence, and may not work well [6].

To determine the type of control chart for each of the qualitative characteristics of the study sample size must be determined. Due to the limited size of the sample for each sample, five, twenty-six times and each sample was taken 5 times. Since all three parameters are the hallmarks of our little bit of charts to use them. The following formula is used to determine the type of a chart. [1] "In general, the sample size for the diagram (\overline{X} , R) and 4 or 5 and the diagram (\overline{X} , S) and at least 10 and to chart P-CHART or np a way that np greater than 3 or 4, so that $15\overline{C} \geq$ is a diagram or U C- CHART." Because we have small sample and the characteristics of our sample size is equal to 5, we charts (\overline{X} , R) and for charting control is used.

There are generally two phases in statistical process control (SPC) applications. In Phase I, a historical set of data is considered to determine the in control process performance and understand the variation in the process over time. In Phase II, actual process monitoring is performed based on the control chart constructed in Phase I.

The general assumption is that the data are normally and independently distributed with mean and standard deviation when the process is in control. If this assumption is violated, the control charts are effected by the violation of independence, and may not work well.

Sampling

To characterize the density of the samples, fat and acidity, respectively, in Tables 1, 2 and 3 are given.

The normal test of observation

Before the calculation and control charts should be drawn from the normal distribution of observations for sure. For this purpose we use Minitab normal test [20] three tests can be done in this area.

Here are the three characteristics of the Anderson-Darling test is performed which results in the form of (1) and (2) and (3) years. In each diagram, the corresponding P-value less than 0.005 after three normal and we can continue to make and implement [21].

Draw a control charts for univariate attributes

In the case of three main characteristics that affect the density, fat and acidity were studied to investigate and develop control charts after sampling, normal distribution function observations were investigated and determined that all three parameters were normal, so it can gain some control, gain control charts. Control charts for attributes density, fat and acidity obtained using Minitab software. And the results are shown in the following figure. First R diagram must be checked and, then the diagram $\bar{\chi}$ is presented [1].

Density control charts:

Control chart in Figure 4 is for the density range as can be seen, there is a point beyond the control and may

be related to the average control chart is plotted in Figure 5. It can be seen and this figure is under control from

these figures, it can be used as control charts are fixed to variable density.

Table 1: Sampling for density

n			Observation	1. 54111511119	Tor derioity	Average	R	S
1	30	30	30	29.5	30	29.9	0.5	0.223607
2	30	30	30	29	29.5	29.7	1	0.447214
3	31	31	29.7	29.5	30	30.24	1.5	0.71624
4	30	30	31	29.9	30	30.18	1.1	0.460435
5	29	29.4	30	29.5	30	29.58	1	0.426615
6	29.5	29	30	30	30	29.7	1	0.447214
7	29.5	29.8	30	30	31	30.06	1.5	0.563915
8	30	30	30	29.9	30	29.98	0.1	0.044721
9	29.5	29.5	30	30	30	29.8	0.5	0.273861
10	29	29.6	30	31	30	29.92	2	0.729383
11	29.8	29.5	30	30	31	30.06	1.5	0.563915
12	29.5	29.6	31	30	30	30.02	1.5	0.593296
13	30	30	29.8	31	30	30.16	1.2	0.477493
14	30	30	29.8	30	30	29.96	0.2	0.089443
15	29	29.5	30	29.9	30	29.68	1	0.432435
16	29	29.4	30	30	30	29.68	1	0.460435
17	29.5	29.5	30	30	29.9	29.78	0.5	0.258844
18	29	29.4	30	30	30.5	29.78	1.5	0.584808
19	30	30	30	29.5	30	29.9	0.5	0.223607
20	30	30	30	31	31	30.4	1	0.547723
21	30	30	29.5	31	30	30.1	1.5	0.547723
22	30	30	29.8	30	31	30.16	1.2	0.477493
23	29.5	29.8	30	30	30	29.86	0.5	0.219089
24	30	30	29.5	31	30	30.1	1.5	0.547723
25	29	29.4	30	30	30	29.68	1	0.460435
26	30	30	29.6	29	30	29.72	1	0.438178
						29.92692	1.030769	

 $\bar{\bar{x}}$ =29.92692; \bar{R} =1.030769

Table 2: Sampling for fat

n			Observation			Average	R	S
1	2	2	2	2.2	2	2.04	0.2	0.089443
2	2	2	2.2	2	2	2.04	0.2	0.089443
3	3	2.7	2.7	2.3	2.1	2.56	0.9	0.357771
4	2.6	2.6	2.4	2.1	2.3	2.4	0.5	0.212132
5	2	3	2.3	2.1	2.1	2.3	1	0.406202
6	2.8	2.6	2.4	2.1	2.1	2.4	0.7	0.308221
7	2.8	2.6	2.4	2.1	2.1	2.4	0.7	0.308221
8	3	2.8	2.3	2.2	2.2	2.5	8.0	0.374166
9	1.6	2.5	2.2	2.2	2	2.1	0.9	0.331662
10	1.9	1.88	2.3	2.1	2.1	2.056	0.42	0.172279
11	2.5	2.5	2.2	2	2	2.24	0.5	0.250998
12	3	2.8	2.3	2.1	1.99	2.438	1.01	0.441837
13	2.5	2.5	2.2	2.2	2.1	2.3	0.4	0.187083
14	2.7	2.5	2.5	2.2	2	2.38	0.7	0.277489
15	2.5	2.5	2.3	2	2	2.26	0.5	0.250998
16	2.4	2.4	2.2	2.2	2.2	2.28	0.2	0.109545
17	2	2	2	2	2	2	0	0
18	2.3	2.4	2.2	2	2	2.18	0.4	0.178885
19	2.5	2.5	2.3	2.3	2	2.32	0.5	0.204939
20	2.5	2.5	2.3	2.3	1.99	2.318	0.51	0.208854
21	1	1.3	2	2	2	1.66	1	0.477493
22	1.1	1.5	1.3	1.25	1.3	1.29	0.4	0.143178
23	2.3	2.3	2	2	2	2.12	0.3	0.164317
24	2.8	2.65	2.41	2.3	2.3	2.492	0.5	0.223763
25	2.5	2.5	2.1	2.1	2.1	2.26	0.4	0.219089
26	2.5	2.5	2	2	2	2.2	0.5	0.273861

 $\bar{\bar{x}}$ =2.212846; \bar{R} =0.543846



21

Table 3: Sampling for PH

	Tuble 5. Sumpling 101 1 11										
n			Observation			Average	R	S			
1	14	14	13.5	14	14	13.9	0.5	0.223607			
2	13.5	14.5	13.5	14	13.5	13.8	1	0.447214			
3	13	13	13.5	14	14	13.5	1	0.5			
4	13.5	13.5	14	14	13.5	13.7	0.5	0.273861			
5	14	14	13.5	14	14	13.9	0.5	0.223607			
6	14	14	14	13.5	14	13.9	0.5	0.223607			
7	13.5	13.5	13.5	14	14	13.7	0.5	0.273861			
8	13	14	13.5	13	13.5	13.4	1	0.41833			
9	14	13.5	14	14	14	13.9	0.5	0.223607			
10	14	14	14	14	13.5	13.9	0.5	0.223607			
11	14	14	13.5	13.5	14	13.8	0.5	0.273861			
12	14	14	14	14	14	14	0	0			
13	14	14	14	13.5	14	13.9	0.5	0.223607			
14	13.5	13.5	14	14	14	13.8	0.5	0.273861			
15	14	14	13.5	14	14	13.9	0.5	0.223607			
16	14	14	14	13.5	14.5	14	1	0.353553			
17	13.5	13.5	14	14	13.5	13.7	0.5	0.273861			
18	14	14	14	14	14	14	0	0			
19	14.5	13.5	13.5	14	14	13.9	0.5	0.41833			
20	14	14	14	13.5	14	13.9	0.5	0.223607			
21	14	14	13.5	14	14	13.9	0.5	0.223607			
22	13.5	13.5	14	14	14.5	13.9	1	0.41833			
23	14	14	13.5	13	14	13.7	1	0.447214			
24	14	14	14	14	14	14	0	0			
25	13.5	13.5	14.5	14	14	13.9	1	0.41833			
26	14	14	13.5	13	14	13.7	1	0.447214			

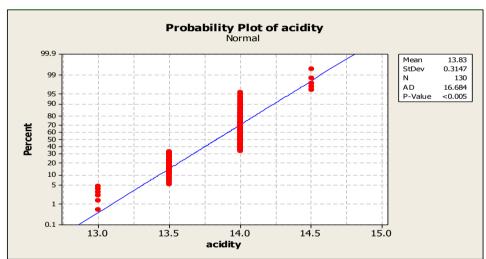


Figure 1: normality test for acidity

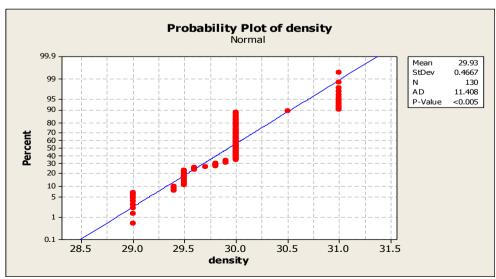


Figure 2: Test of normality for the density

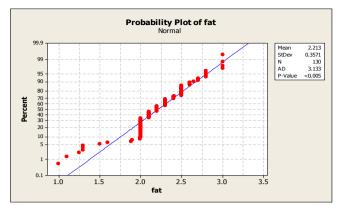


Figure 3: Test of normality for fat

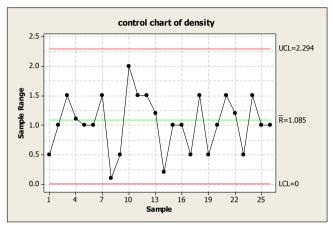


Figure 4: Diagram of density control range

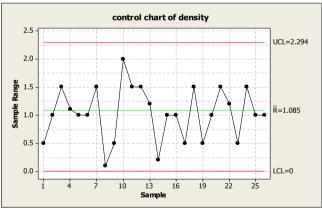


Figure 5: Mean density chart

Fat control charts:

The range for this variable control chart in Figure 6, and the point is out of control, the average control chart (Figure 7) is checked Figure (7) shows that the figure four points outside our control. As indicated in Figure 21 and 22 points out of the area 3σ than the central line and points 7 and 8 with regard to the four points of the 5-point distance 1σ from the center line (on one side of the center line) are outside their control [1].

Examining this point we realize that outside the control of the operator due to a technical problem or improper use of the separator (separator) respectively. So it can be eliminated and the new control charts to draw from. Since the removal of the 4 points to 26 points again,

the number of points (22 points) more than 20 new sample is needed [1]. The range of changes in fatcorrected diagram shows in Figure (8). Figure (8) of the points 21 and 22 continue with 9 consecutive points on one side of the central line [1], show is out of control charts, so we're not allowed to examine the average control charts and graphs must modify the changes. With the elimination of 21 and 22, because the number of observations we have not less than 20 [1] Changes in the scope of the new diagram in Figure (9) draw. With regard to the form (9) observed that all 20 are under control charts therefore, we examine the average control chart as shown in Figure 10, figure out the controls are the reference charts to assess lipid parameters as well as the two graphs are shown in Figures 9 and 10.

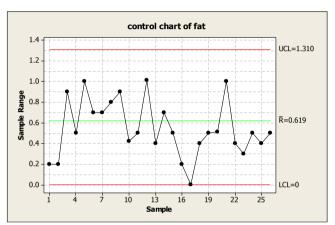


Figure 6: Graph control the fat

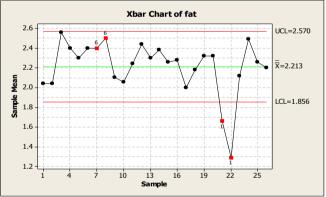


Figure 7: Mean control chart for fat

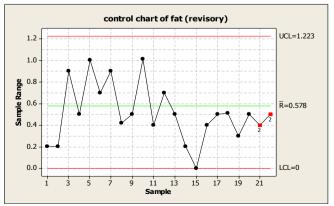


Figure 8: Trim the fat chart

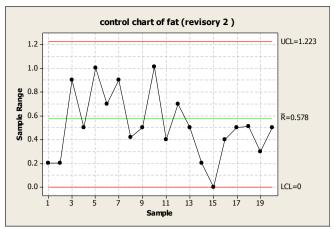


Figure 9: Graph modified second domain controller for fat

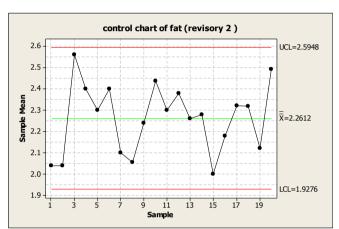


Figure 10: Mean control chart for fat

Control charts for PH:

The range for this variable control chart in Figure 11, and the point is out of control, the average control chart (Figure 12) is checked with regard to the form (12) observed an out of 8 samples from the center line [1], so the process is out of control. 8 is a check point operators have difficulty in adjusting device and as we know, the water content in the product to influence the amount of acidity, the result can be removed and the new control point is plotted.

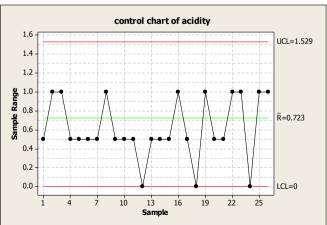


Figure 11: Diagram to control the acidity

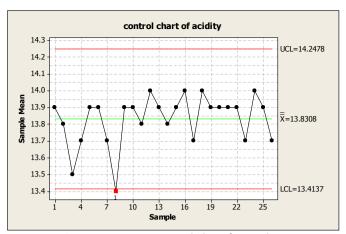


Figure 12: Mean control chart for acidity

Figure 13 is a diagram of the new domain controller. 13 and 14 points out of control and because 11 consecutive points on one side of the center line are out of control charts [1], the average control charts can be reviewed and removed the three points, the new control chart in Figure 14 are plotted. With regard to the form (14) diagram is corrected for the change in control, so we can control chart for the mean as in (15) we drew. With regard to the form (15) is observed the average figure is well under control the charts also form the basis for the acidity (14) and (15) respectively.

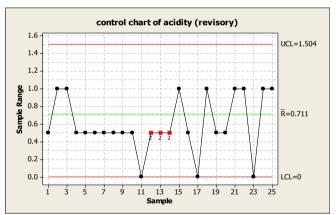


Figure 13: Graph modified to control the acidity

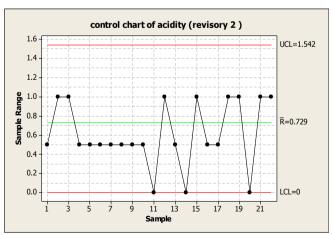


Figure 14: Diagram of the second domain controller for pH correction

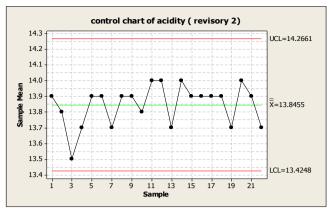


Figure 15: Mean control chart for acidity

Drawing control charts for Multivariate attributes

Using MINITAB software, which included the following, it is clear that there is a correlation between the quality characteristics. Because of the correlation between the use of Chart Hotelling seems necessary [1], thus, multivariate Hotelling's chart is used to control the process.

Correlations: acidity, density, fat acidity density density -0.011

0.901

fat -0.167 -0.108 0.058 0.223

Cell Contents: Pearson correlation

P-Value

Based on the above results that the values of the correlation coefficient for each pair with the P-Value indicates we see that there is a correlation between these values, because none of the coefficients obtained is zero or a value close to zero. Hotelling control chart for three characteristics of fat, density, acidity, depend on each other to form (16).

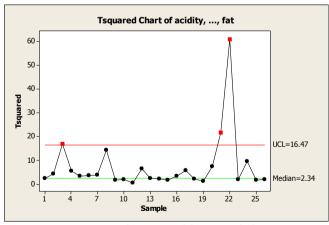


Figure 16: three-variable control chart

Figure 16 shows a diagram of the 3, 21 and 22 are out of control. To paraphrase a point out of control in this type of diagram, we deal with the problem. In fact, we are confronted with the question if an out of control to be drawn, and then which of the following categories are causing this warning? The answer to this question is not always simple. Here we examine the software and the

individual diagrams are drawn in the last section, we use [1]. 21 and 22 points out of control are due to improper use or operation of the separator, the separator. For the third because in the early stages of sampling and not completely out of the control of a situation occurred, thus eliminating the three categories above, we can see the control limits are modified according to the form (17) is reached. We see that all of the new control charts are controlled by and it can be used as a base curve.

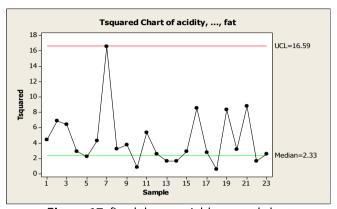


Figure 17: fixed three-variable control chart

CONCLUSION

In the process of analyzing three main characteristics that affect the density, fat and the acidity of the milk product for review and preparation of control charts were investigated. After sampling, samples of normal distribution function, examined we the three characteristics were normal and so we can get out of control charts. Control charts for attributes that are then plotted and after checking out of the process control and radical reform as univariate control charts were fixed. Given the correlation between the characteristics of Hotelling multivariate control chart for process tree vitiate control were drawn. And it was observed that out of control in the first one characteristic curves for the mean data for lipid parameters and a pH of 7, 8, 21, 22 hotelling, three characteristic for samples 3, 21, 25, respectively. This indicates that the correct analysis and review process under its control has better characterization of single and three-variable control charts can be used simultaneously.

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