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Prediction Using a Hybrid Particle Swarm Optimization and GARCH Model

Reza Narimani[1*] and Ahmad Narimani[2]

[1] Department of Financial Engineering, University of Economic Sciences, Tehran, Iran
[2] Department of Economics, University of Allameh Tabatabae'i, Tehran, Iran
*Corresponding author's email: Reza_narimani@yahoo.com

Abstract

This paper addresses one of the most important data mining tasks, forecasting, where the objective is to predict any time series. In order to achieve the best performance. At the first stage, we find the best GARCH model by looping through AR and MA term. At the next stage, we propose PSO-GARCH model that optimize the coefficient of the model. Particle swarm optimization is one of the best optimization techniques that is suitable for continues variable, thus by using PSO we can find the best coefficient of GARCH model obtained from first stage. The fitness function gets the weights from PSO particle and optimizes the weights by using Marquardt algorithm. In our experiment, two dataset were used: The daily observations of Brent oil price and the exchange rate of US Dollar to Euro (USD2EUR). The achieving results show that the performance of the hybrid method is so promising.

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INTRODUCTION

Forecasting volatility in financial markets is one of the very important issues that has attracted many researchers and experts in this field during the past decades. This is important since the volatility in the financial market is one of the important variables in investment decisions, pricing of securities and derivatives, risk management, regulation and monetary policy. In addition, volatility in financial markets has an important impact on the economy through the creation or loss of public confidence and trust.

ARIMA models assume a linear relationship between the current value of the underlying variables and previous values of the variable and error terms. Financial Time series models are highly nonlinear and the mean and variance of the series can change overtime [1]. In order to overcome this difficulty, an autoregressive conditional volatility models (ARCH) that originally was introduced by Engle [2] and this model is generalized later by Bollerslev [3] and now it is an important model for the analysis of high frequency financial time series data.

Autoregressive conditional heteroscedasticity (ARCH) models have been widely used in financial time series analysis and particularly in analyzing the risk of holding an asset, evaluating the price of an option, forecasting time-varying confidence intervals and obtaining more efficient estimators under the existence of heteroscedasticity [4].

In the context of volatility prediction in the exchange, numerous studies have been carried out using these models. For example, in [5] showed that the ARCH and GARCH models, exponential weighted moving average models in predicting the average historical volatility of monthly U.S. stock index has better performance.

The study in [6], investigate the conditional heteroscedasticity and distribution type of stock returns in the Taiwan stock market. The results show that the GARCH (1,1) model with mixture of normal distribution is better model for the Taiwan stock returns.

The study in [7], use linear models and GARCH model to forecast two stock indexes in China stock market. The results show that depending on the criteria, the prediction models are different, but the overall performance of random walk model is worse than all other model. Because of the conditional variance in GARCH model, this model has been used more than ARIMA model in finance. The application of GARCH model in estimation value at risk (VaR) has been worked by many researchers: [8, 9].

GARCH model is composed of a mean equation like ARIMA and has addition conditional variance. Many researchers have been developed based on ARIMA.

ARIMA is kind of models that if it well-defined, it can eliminate the auto correlation and partial correlation between residual. Thus many researchers use this model as a part of hybridization their proposed model. The researchers in [10], use a hybrid model of ARIMA and probabilistic neural network (PNN) in order to yield more accurate results than traditional ARIMA models.

The research in [11] and [12] present an ARIMA model which uses particle swarm optimization algorithm (PSO) for model estimation.

In this paper, we focus on the better estimation of the parameters of GARCH model. Since, this step is a complex process; we adopt particle swarm optimization (PSO) to estimate the parameters. PSO is a population based stochastic optimization technique and thus can reach near global optimum of very complicated function.

This paper is organized as follows: Section 2 reviews the detailed process of GARCH model and PSO algorithms. The definition of the traditional model estimation; Section 3 introduces simple GARCH model estimation and our proposed estimation method, PSO-GARCH; In Section 4, we implement this method on two dataset:1) Brent oil price and 2) the exchange rate of US Dollar to Euro (USD2EUR) to illustrate the excellent forecast ability of this new hybrid model.

2. Forecasting Method

2.1 GARCH model-Generalized ARCH: GARCH model was originally designed and introduced by Bollerslev [3] and Taylor [13]. They show that the mean and the variance of the model can be simultaneously estimated. The GARCH (1,1) model is a very popular specification because it fits many data series well. It tells us that the volatility changes with lagged shocks (e_{t-1}^2) but there is also momentum in the system working via σ_{t-1}^2. One reason why this model is so popular is that it can capture long lags in the shocks with only a few parameters [14]. In this paper the mean equation is an ARIMA process and the variance of model is a GARCH (1,1):

$$\phi(\beta)\nabla^d(y_t-\mu)=\theta(B)u_t \quad u_t \sim N(0,\sigma_t^2) \tag{1}$$

$$\sigma_t^2=\alpha_0+\alpha_1 u_{t-1}^2+\alpha_2\sigma_{t-1}^2 \tag{2}$$

The conditional variance in GARCH (1,1) dependent on previous own lag ($\alpha_2\sigma_{t-1}^2$) and volatility during previous period ($\alpha_1 u_{t-1}^2$).

The log-likelihood function (LLF) based on normality assumption is:

$$L=-\frac{T}{2}\log(2\pi)-\frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2)-\frac{1}{2}\sum_{t=1}^{T}(y_t-\hat{y}_t)^2/\sigma_t^2 \tag{3}$$

Where T is sample size and $\hat{y}_t$ is estimation of y_t.

One constraint of the model comprises of the fact that the unconditional variance of u_t is constant and given by

$$\mathrm{var}(u_t)=\frac{\alpha_0}{1-(\alpha_1+\alpha_2)} \tag{4}$$

In the above equation $\alpha_1+\alpha_2\le 1$ must be held to unconditional variance is defined. Estimation of parameter of GARCH model can be implemented by [15]. approach that has three stages: 1) identification 2) Estimation 3) Diagnostic checking.

Identification can be held by minimizing the information criteria. Information criteria are composed by two terms: 1) a function of residual sum of square (RSS); 2) penalty for the loss of degree of freedom. The most information criteria that is recommended is Akaike's information criteria that is given by

$$AIC=\log(\hat{\sigma}^2)+\frac{2k}{T} \tag{5}$$

Where $\hat{\sigma}^2$ is the residual variance, k is total number of parameters estimated and T is the sample size.

2.2 Estimation of GARCH models

Since the GARCH model is not in linear form, OLS cannot be used for GARCH model estimation. One of the methods to estimate the parameter is maximum likelihood. But there is the problem of local optima in maximum likelihood estimation. Local optima or multimodalities in the likelihood surface present potentially serious drawbacks with the maximum likelihood approach to estimating the parameters of a GARCH model [16]. Thus, in this paper we use a PSO-GARCH model to overcome this problem.

2.3 Particle swarm optimization (PSO) algorithm

Originally, Particle Swarm Optimization was proposed by [17]. The main idea of PSO is social behavior of birds, bees or a school of fishes. In PSO algorithm, each particle can move along the linear combination of its own velocity, towards best global position and towards best local of its own position in the problem space. The velocity of a particle is updated from Eq.6 and Eq.7.

$$v_{ij}(t+1)=w\,v_{ij}(t)+r_1C_1\left(P_{ij}(t)-x_{ij}(t)\right)+r_2C_2\left(g_j(t)-x_{ij}(t)\right) \tag{6}$$

$$x_{ij}(t+1)=x_{ij}(t)+v_{ij}(t+1) \tag{7}$$

Where w is inertia weight that shows the effect of previous velocity on new velocity vector. C1 and C2 are positive constant, r1 and r2 are random variables with uniform distribution in between 0 and 1.

The constraint for new velocity vector is given by:

$$\left|v_{ij}(t)\right|\le V_{max} \tag{8}$$

$$V=\alpha(x_{max}-x_{min}) \tag{9}$$

The value of 0.1 for α is appropriate. If x is out of bound, then Eq.10 must be computed for new position

$$x_b=\min\{\max(x,x_{min}),x_{max}\} \tag{10}$$

2.4 The proposed model

Our model consists of three stages: At the first stage, the order of AR and MA term can be differed from 1 to 5. Hence the total search space is equal to 210 states. The objective that must be minimized is the Akaike's information criteria in Eq. 5. Basic GARCH has the search space of AR and MA term is started from 1 to 5 without any omitted order. This search is shown in Table 1.

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Table 1. Algorithm for estimation GARCH model

1. **For** $p = 1$ *to* 10 **do**
2. **For** $q = 1$ *to* 10 **do**
3. $consider: \begin{cases} \phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \\ \theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q L^q \\ \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \end{cases}$
4. *estimate parameterby MaximizingLLF*
5. *compute AIC (Akaike's Information Criterion*
6. **endFor**
7. **endFor**
8. *select the* model *with lowest AIC*

At the second stage with the best AR and MA term obtained from stage one, we implemented a particle swarm optimization to reach better coefficient.

In this paper we set the parameter of PSO that is recommended in [18] which reaches a set of coefficients to control the system's convergence tendencies to increase the ability of the particle swarm to find optima of some well-studied test functions. This coefficient along the other parameter of PSO is given by Table 2:

Table 2. The parameters of the PSO

	Parameter	Level 1
Inertia weight	w	0.7298
The weighting of the stochastic Acceleration terms	$c_1 = c_2$	1.4962
Population size	n_{pop}	30
Maximum iteration	it_{Max}	20
Maximum values for The positions	$\mathrm{var}_{\max}$	1
Minimum values For the positions	$\mathrm{var}_{\min}$	$-\mathrm{var}_{\max} = -1$
Maximum values For the velocities	$val_{\max}$	$0.1 \times (\mathrm{var}_{\max} - \mathrm{var}_{\min}) = 0.2$
Minimum values For the velocities	$val_{\min}$	$-val_{\max} = -0.2$

Each particle presents the weights of mean and conditional variance. A particle is constructed from a series of weights as shown in Fig.1. In this Figure, for a GARCH model that has the order p of AR term and q of MA term is shown. The connection weights are real numbers.

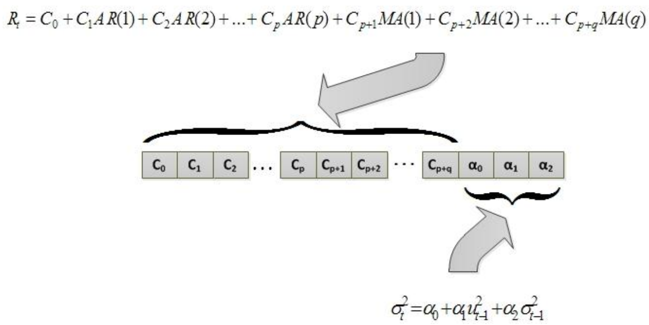

Figure 1. Particles in PSO-GARCH

So with the initial solution, we optimize the coefficient of GARCH by Marquardt algorithm. The maximum iteration to optimize is set to 30 iterations. The main reason for choosing 30 iterations is that by very poor initial solution, this number of iterations is a tool that gets this initial solution to the better solution. This stage is shown on Fig. 2.

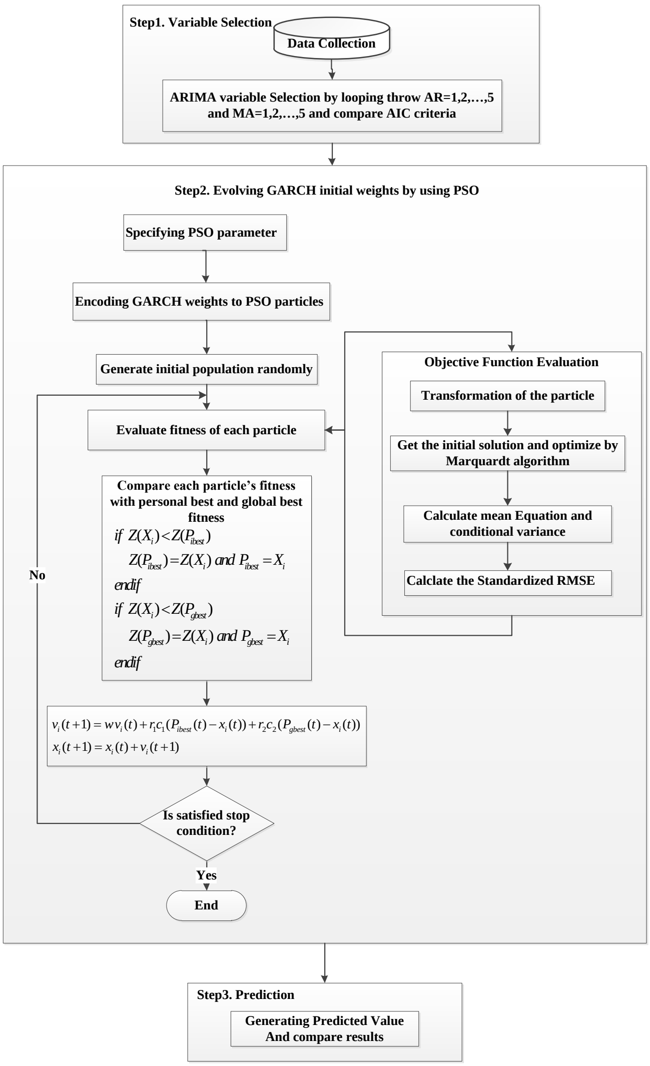

Figure 2. Framework of PSO-GARCH model

The more detail of fitness function is shown on figure. We set the constant in mean equation divided by 100. The parameters of conditional variance are first mapped between 0 and one. Then, the constant in variance equation is divided by 30. If $\alpha_1 + \alpha_2 > 1$ then the unconditional variance is not defined. So, we use this simple transformation to avoid this situation $\alpha_2 = (1 - \alpha_1) \times \alpha_2$). By this transformation α_1 is between 0 and 1 and the reaming amount to 1 is allocate to α_2 by the proportion of it.

One important issue is the fitness function in PSO. The best choice is –LLF or AIC or RMSE of standardized error. We set RMSE of standardized error ($\sqrt{\frac{1}{T} \sum_{i=1}^{T} \left(\frac{y_i - \hat{y}_i}{\sigma_i} \right)^2}$)

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as the objective function that must be minimized. The algorithm for fitness function is shown on Fig. 3. Another important matter is that if the estimated AR or MA process is non-stationary then a death penalty is considered. In this stage we use particle from PSO as initial solution for GRACH model.

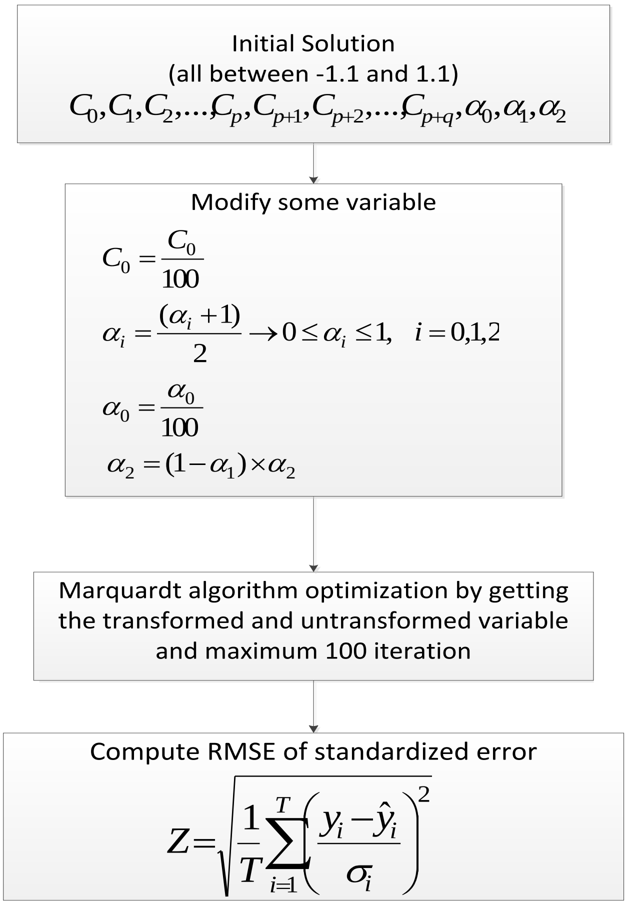

Figure 3. The fitness function of PSO-GARCH model

At the third stage, the final model obtained from stage two compared with the basic GARCH model. The performance factor that used to compare is described in section 2.3 and the results is described in section 3.

2.5 Model performance evaluation

To determine the performance of each model, two different criteria were implemented: 1) the root mean square error (RMSE) and mean absolute percent error (MAPE) that can be computed as:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (\hat{y}_i - y_i)^2}{N}} \tag{11}$$

$$\mathrm{MAPE} = 100 \sum_{i=1}^{N} \frac{\left| \frac{\hat{y}_i - y_i}{y_i} \right|}{N} \tag{12}$$

Where N represents the number of test data and $\hat{y}_i$ is forecasted value from the model and y_i is actual value in *i*th test data.

We choose two dataset for evaluating the proposed model and compare the result with the basic GARCH model and PSO- GARCH model. The first dataset is Brent Oil Price (BP) over the period 9 April 2012 – 4 Jun 2012 and the second dataset is USD2EUR over the period 28 April 2012 – 7 Jun 2012. Each dataset covers 41 daily observations that 35 daily data is used for training and 6 data used for testing. A graph representation of these data is shown in Fig. 4:

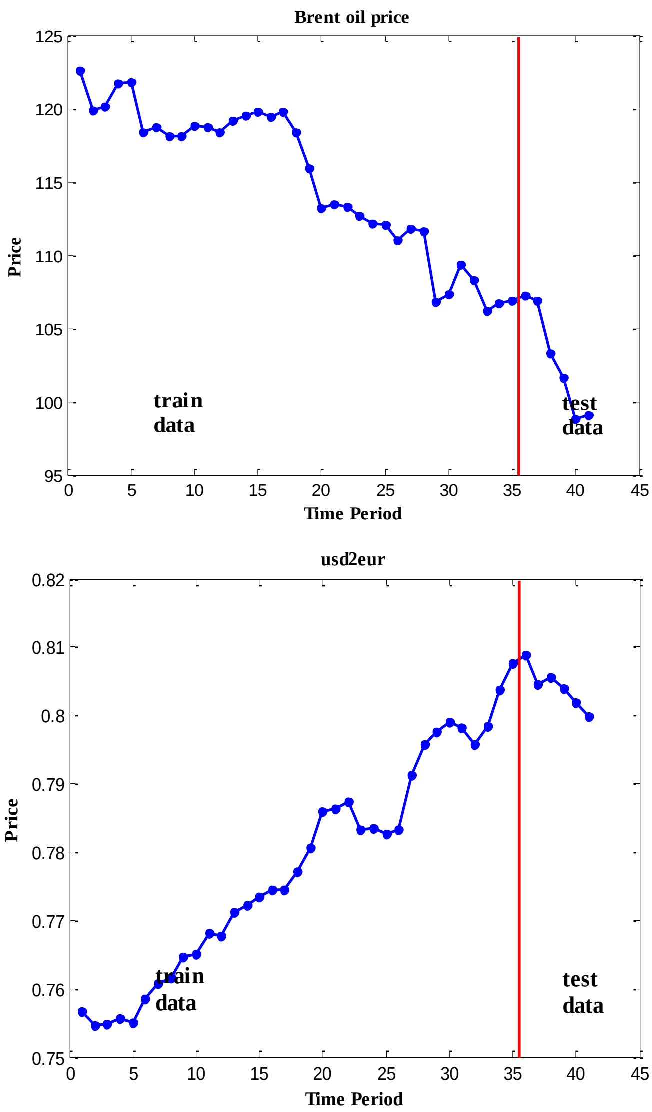

Figure 4. The pattern of Brent oil price and USD2EUR exchange rate

RESULTS

The order of GARCH model for both dataset is shown in Table 3. The value of RMSE for basic GARCH model and PSO-GARCH model of each dataset in training stage is shown on Table 4. To observe the evolutionary process in our model, **Error! Reference source not found.** shows the evolution of the best fitness on the PSO-GARCH where the x axis is the indicator of the number of function evaluation.

Table 3. the order of GARCH model

Order	Brent Oil	USD2EUR
AR	AR(1) AR(2) AR(3)	AR(1)

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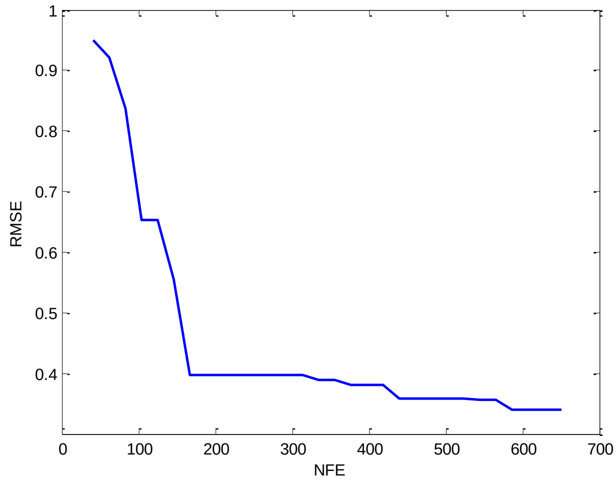
MA	MA(1) MA(2) MA(3)	MA(1) MA(2) MA(3)

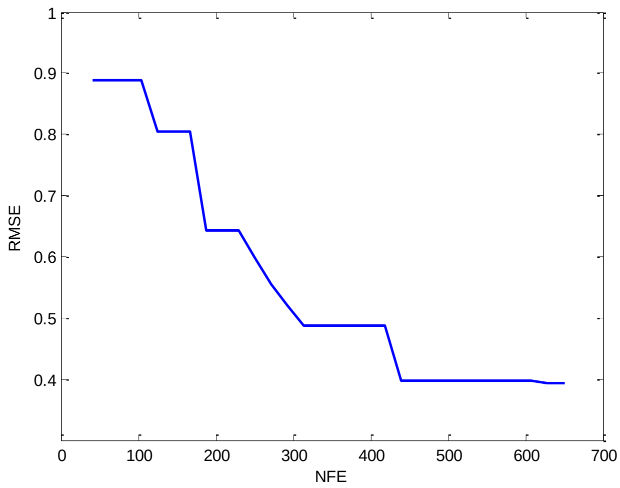
Table 4. The RMSE of the basis GARCH and PSO- GARCH models

Brent Oil		USD2EUR	
Basic GARCH	PSO- GARCH	Basic GARCH	PSO- GARCH
1.0208	0.33962	1.0160	0.39413

Figure 5 shows the evolution of the best fitness in the PSO that estimates the parameter of the model that that obtained by PSO-GARCH.



(a)



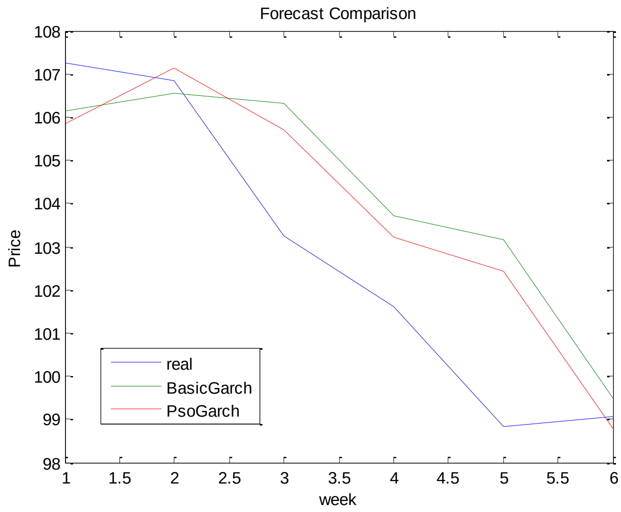
(b)

Figure 5. PSO plot for the best fitness during the training phase on the (a) Brent oil and (b) USD2EUR

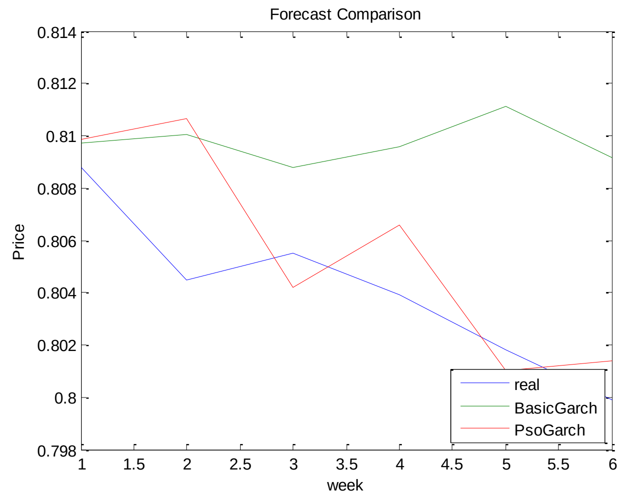
Table 5 gives the forecasting results for the Brent oil and USD2EUR exchange rate. The result of test data is shown on Fig. 6.

Table 5. Forecasting performance of the Basic GARCH and PSO-GARCH and Hybrid model

	Brent Oil		USD2EUR	
Performance	Basic GARCH	PSO-GARCH	Basic GARCH	PSO-GARCH
RMSE MAPE	2.3844 1.8637	1.9960 1.5805	0.0064 0.7068	0.0029 0.2791



(a)



(b)

Figure 6. Forecasted and Test data for (a) Brent oil and (b) USD2EUR

Hence, this hybrid model can converge quickly to near global optima and we can enhance its effectiveness for our hybrid model.

CONCLUSION

The result of the present paper indicates that this hybrid model performs better results than basic GARCH and PSO- GARCH. The forecasting results indicate that the hybrid model is more effective rather than the basic GARCH model.

In this paper, we show the hybridization of the GARCH with PSO. So the traditional models can be a very powerful tool to analyze and estimate the chaotic behavior in financial markets if we combine them with recent optimization method.

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